# An efficient frontier analysis to determine final pit under geological uncertainty

## 2017 INFORMS Annual Meeting

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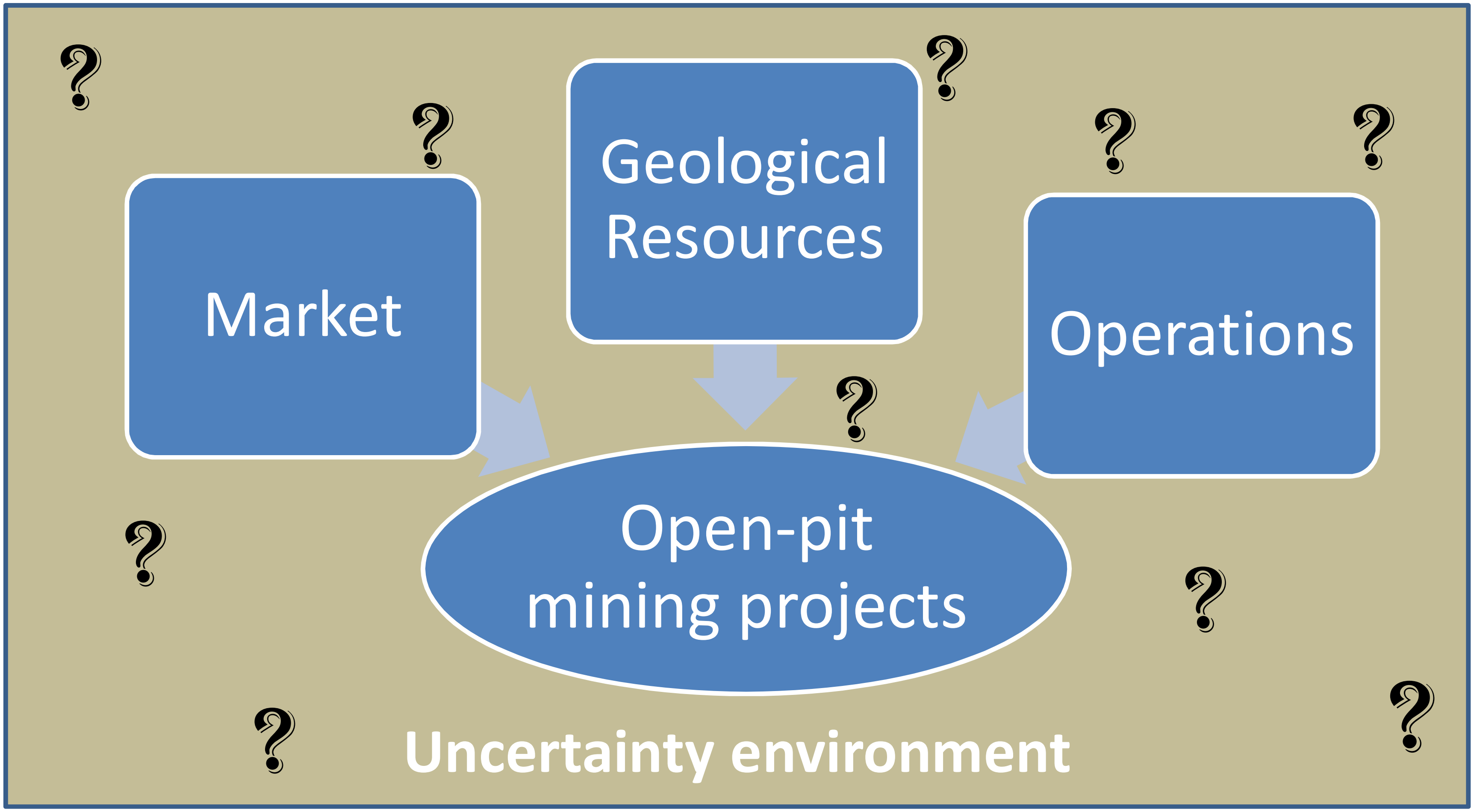
# Outline

- Introduction
- Problem definition
- Review of related work
- Proposed methodology
- Case study
- Results
- Conclusions and future work

# Introduction



# Introduction

**Geological uncertainty**

- Extension and position of GU
- Types of material and their concentrations

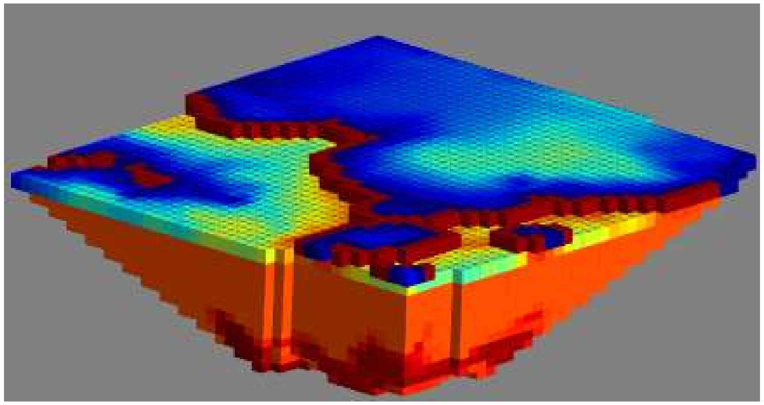
    Example: ore grade (copper)

**How the geological uncertainty is represented?**

- Set of **scenarios (realizations)**
- Condicional simulations: values from CS honor the sample values.

# Problem definition

Deposit → Block model



Which is the subset of blocks that maximize the profit of exploitation satisfying slope precedence constraints ? Pit final

# Problem definition

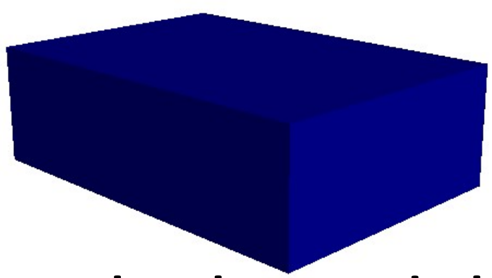
$$x_b = \begin{cases} 1 & \text{if block } b \text{ belongs to final pit,} \\ 0 & \text{otherwise.} \end{cases}$$

Thus, the ultimate pit limit can be found solving the following problem

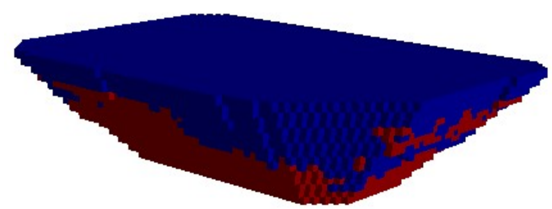
$$\begin{aligned} (\mathrm{P}) \quad \max \quad & \sum_{b \in \mathcal{B}} \bar{v}_b x_b \\ \text{s.t.} \quad & x_b \leq x_{b'} && \forall\, b \in \mathcal{B},\, b' \in \mathrm{PREC}(b) \\ & x_b \in \{0,1\} && \forall\, b \in \mathcal{B} \end{aligned}$$

**Problems?**

- **The final pit decision is taken on a single representation of the deposit.**
- **In-situ geological uncertainty in not take into account.**



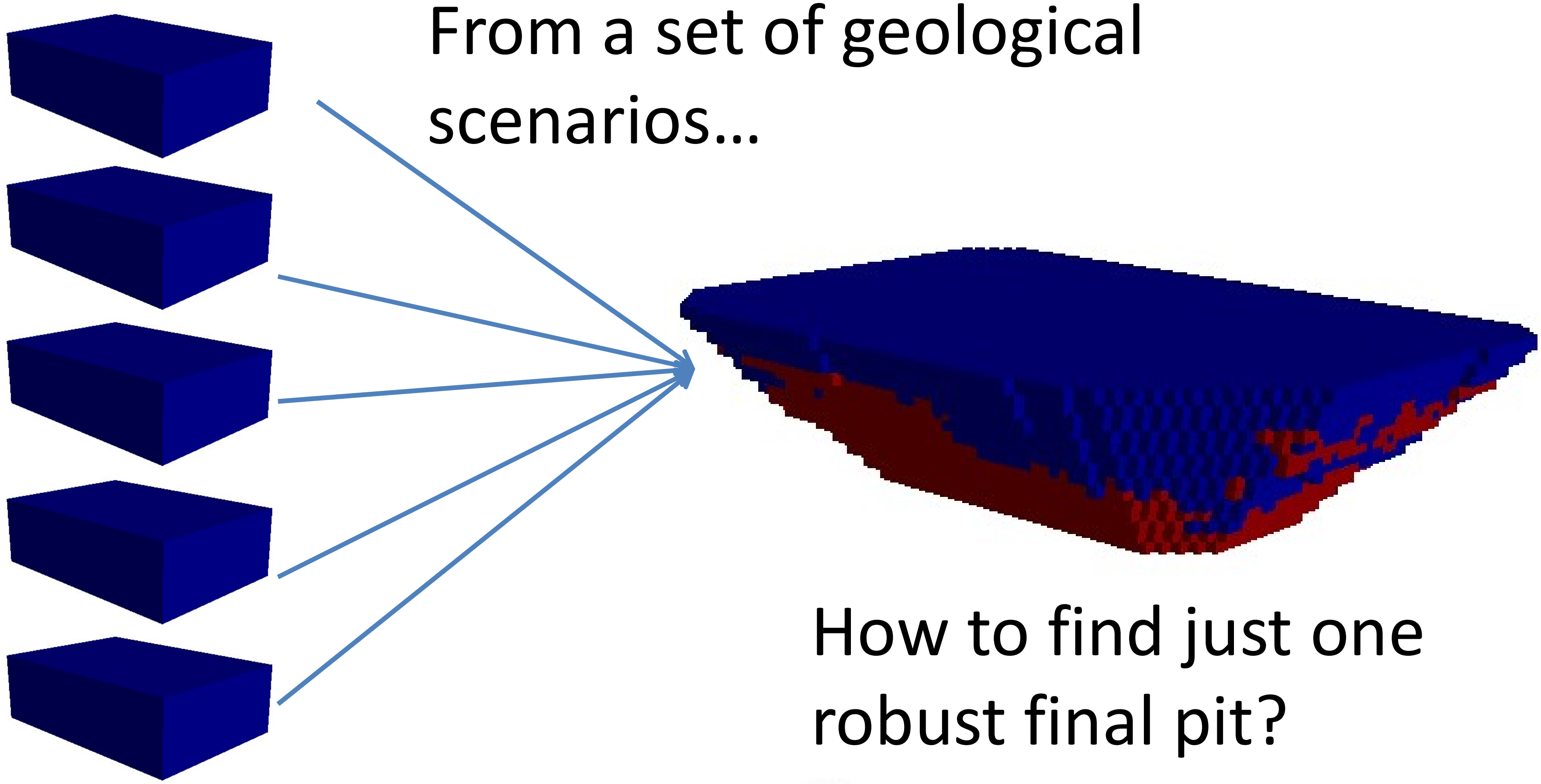
Block model



A final pit

# Problem definition

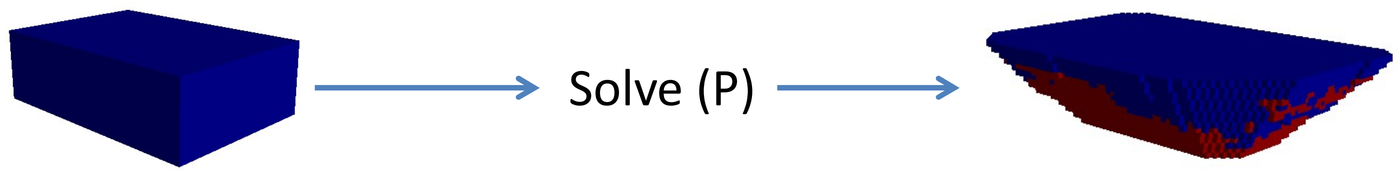
From a set of geological scenarios…



How to find just one robust final pit?

# Review of related work
# Deterministic approach

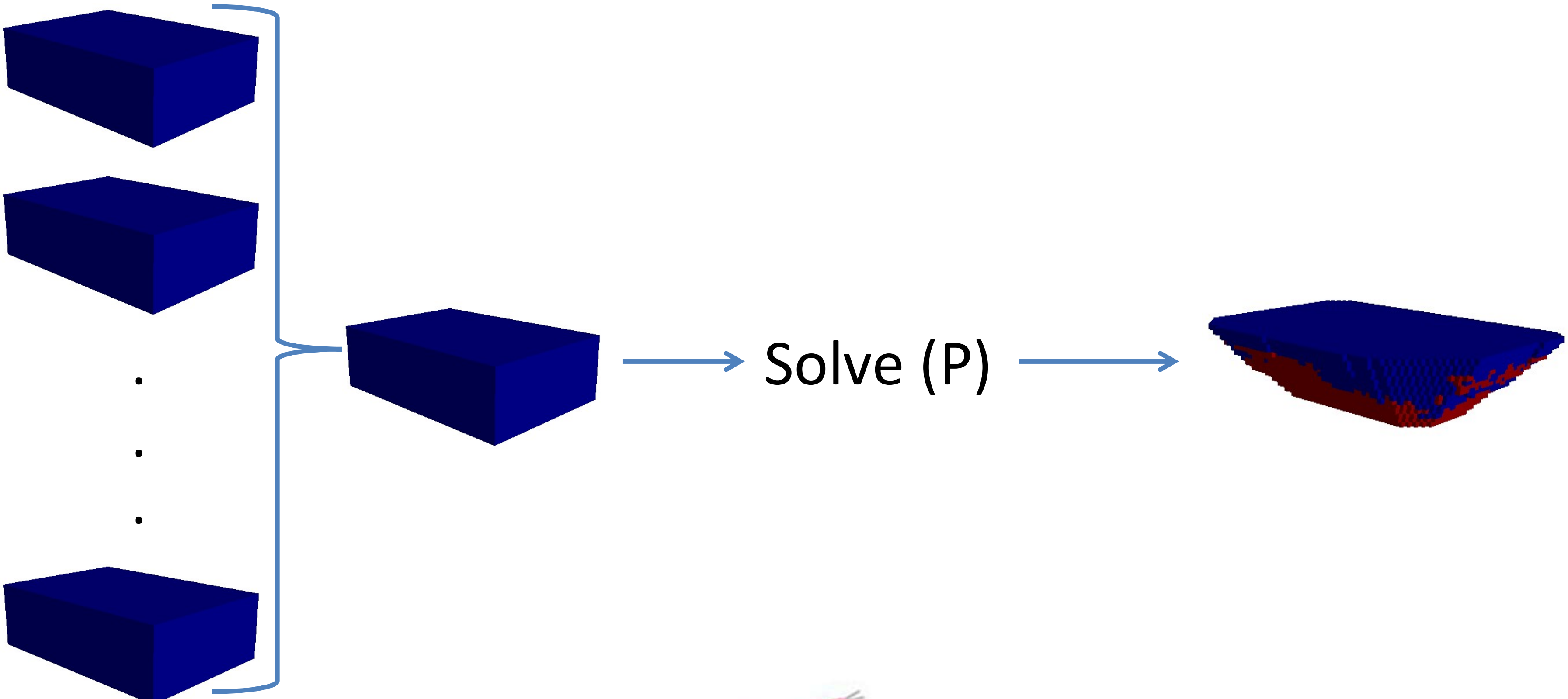
- **One scenario: Etype, Kriging**




This solution can be evaluated along set of N geological scenarios in terms of expected value

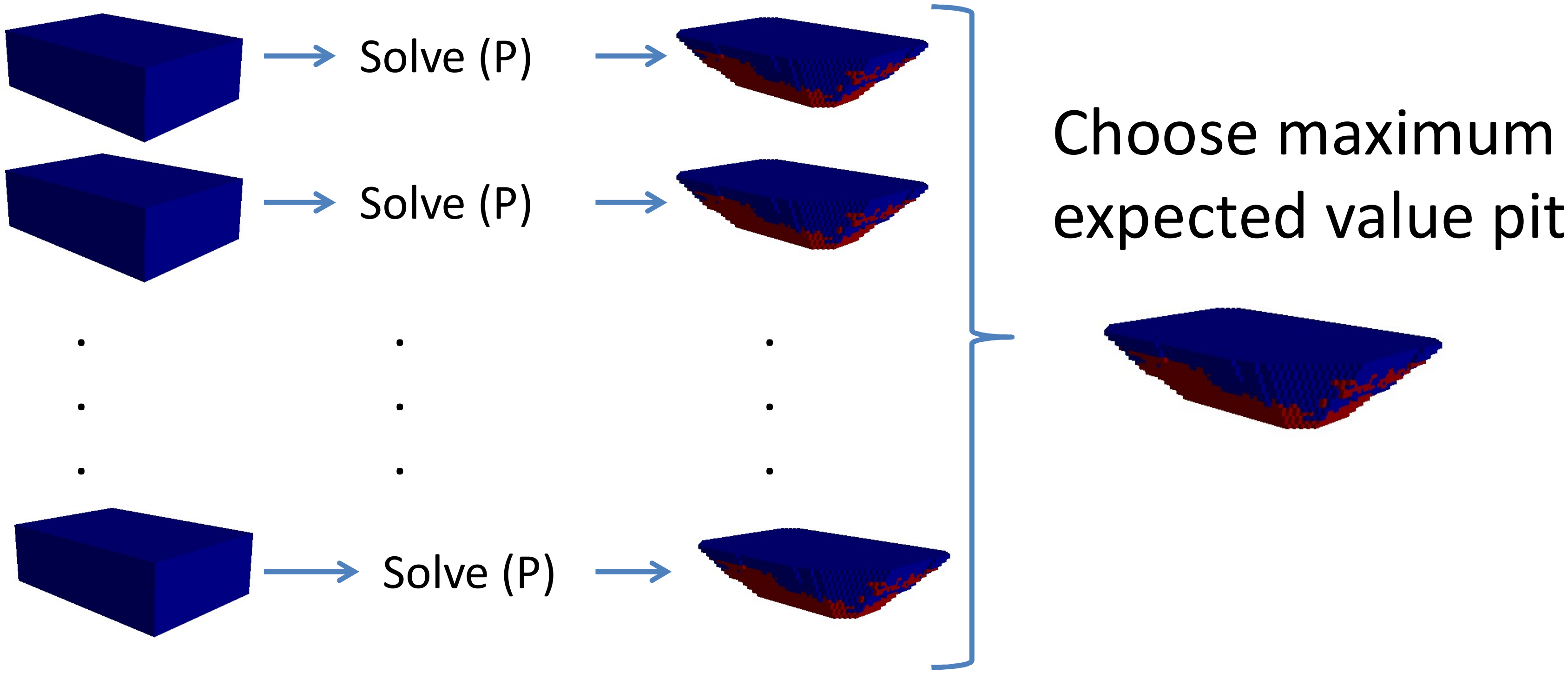
# Review of related work
# Stochastic approach

- **Expected value (Marcotte and Caron, 2013)**



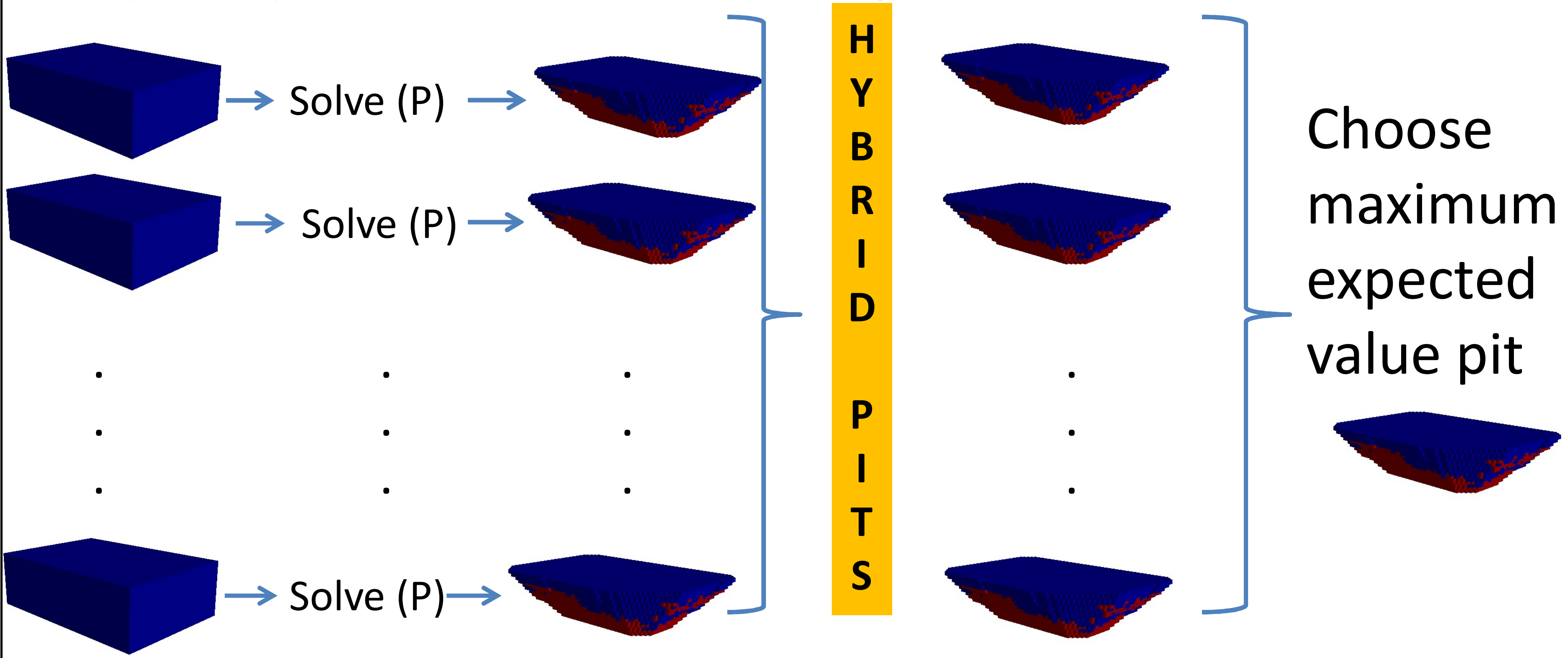
# Review of related work
# Simulation approach

- **Best simulation (Dimitrakopoulos et al, 2007)**



# Review of related work
# Reliability approach

- **Hybrid pits (Whittle and Bozorgebrahimi, 2004)**

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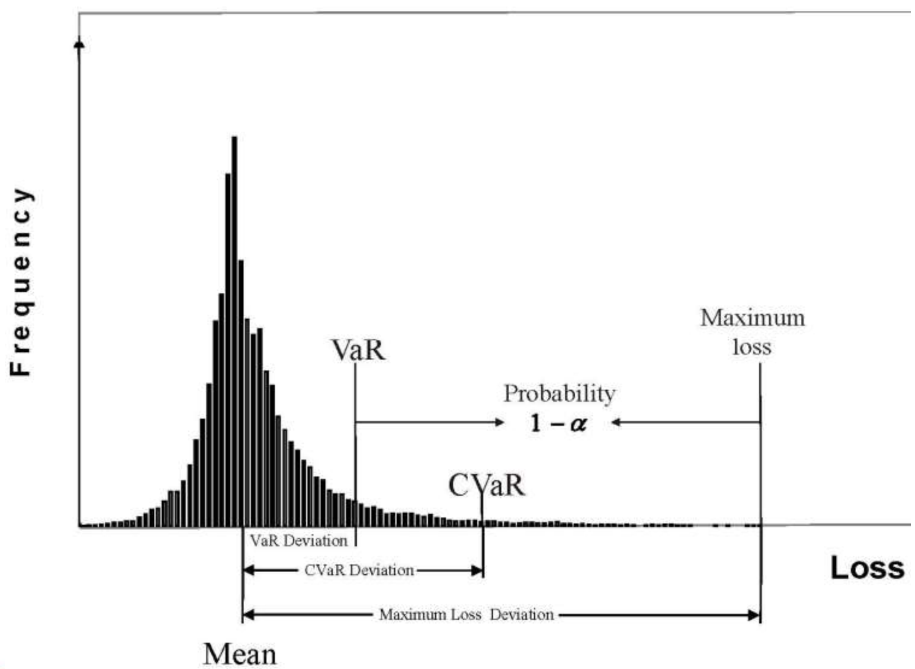
# Proposed methodology

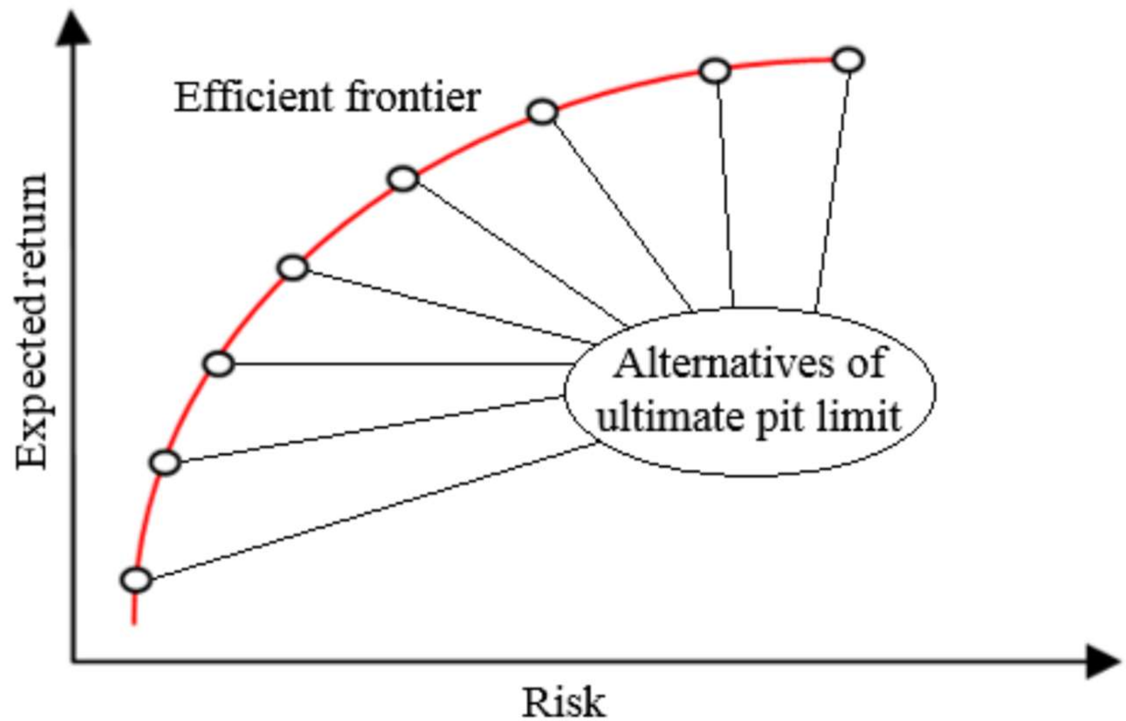
We develop a model to generate the EF of final pits

- $\boldsymbol{VaR_\alpha}$(X): worst loss expected when we remove their (1-α) maximum losses.
- $\boldsymbol{CVaR_\alpha}$(X): conditional expectation of X subject to X ≥ $\mathrm{VaR}_\alpha$(X)



# Proposed methodology

$$\max_{\boldsymbol{x}\ es\ pit} [ExpVal(\boldsymbol{x}) - \mu \cdot CVaR_{\alpha}(\boldsymbol{x})] \quad , \mu \geq 0$$



# Proposed model

**Model**

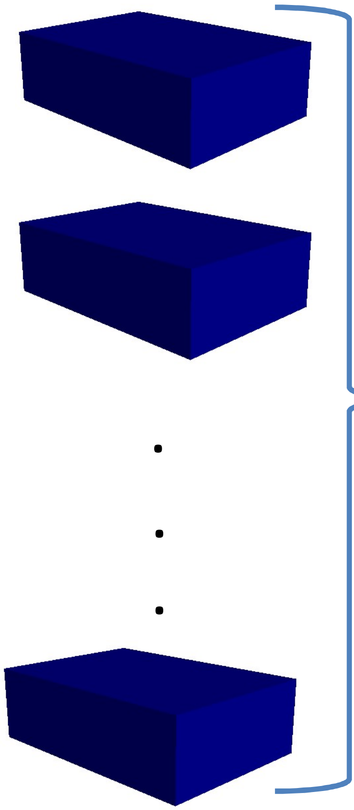
$$(\mathrm{P}_\mu)\ \max\left[\frac{1}{R}\sum_{\substack{r\in\mathcal{R}\\ b\in\mathcal{B}}} v_{br}\,x_b - \mu\left(\zeta + \frac{1}{R(1-\delta)}\sum_{r\in\mathcal{R}} z_r\right)\right]$$

$$\begin{aligned}
\text{s.t.}\quad & x_b \le x_{b'} && \forall\, b\in\mathcal{B}, b'\in \mathrm{PREC}(b)\\
& z_r \ge f(\mathbf{x},\mathbf{y}^r) - \zeta && \forall\, r\in\mathcal{R}\\
& z_r \ge 0 && \forall\, r\in\mathcal{R}\\
& x_b \in \{0,1\} && \forall\, b\in\mathcal{B}
\end{aligned}$$

**Optimal solution**: $(\mathbf{x}^*, \zeta^*, \mathbf{z}^*)$

**Loss function**: $f(\mathbf{x},\mathbf{y}^r) = \sum_{b\in\mathcal{B}} (v_{br} - \bar{v}_b)\, x_b \quad \forall r\in\mathcal{R}$

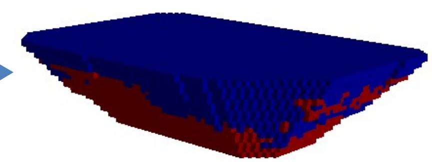
# Proposed model



$$Max[ExpVal(\boldsymbol{x}) - \boldsymbol{\mu} \cdot CVaR_{\alpha}(\boldsymbol{x})]$$

$\boldsymbol{x}$ is a pit

$Solve\ (P_{\mu})$

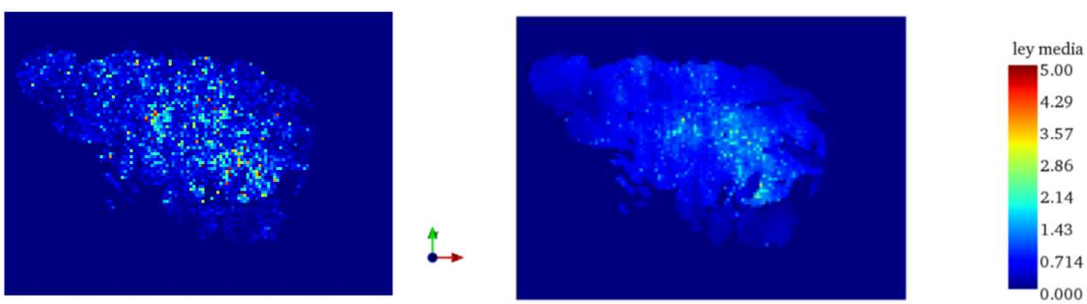


$\boldsymbol{\mu - final\,pit}$

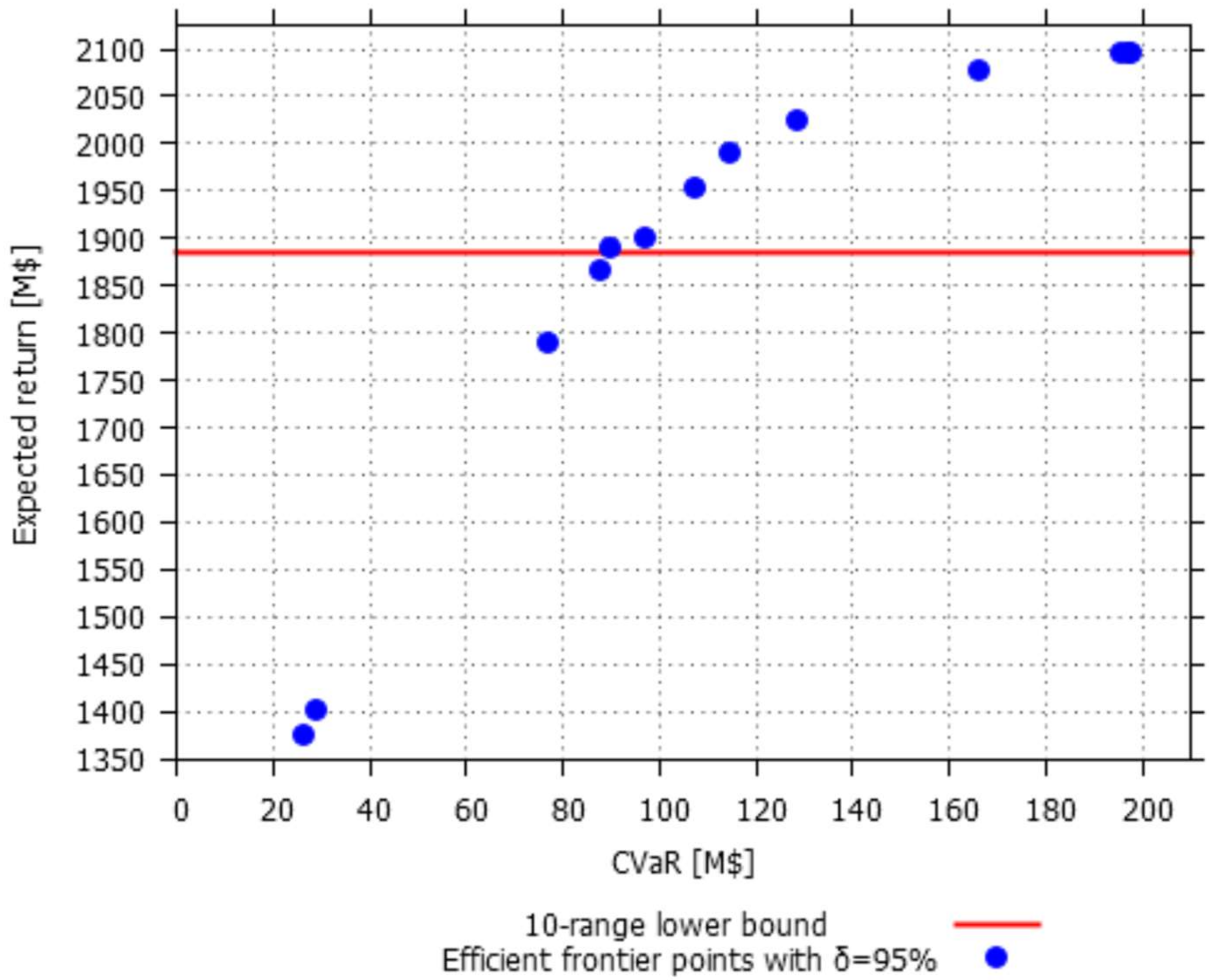
# Case study

- Porphyry copper deposit
- Block model: ~ 410.000 cubic blocks
- 50 conditional simulations (cu grade)

| Parameters | Value or criteria |
|---|---|
| Price (USD/lb) | 2.5 |
| Metallurg. recovery | 0.85 |
| Mining cost ($/ton) | 3.2 |
| Processing Cost ($/ton) | 9.0 |
| Selling cost ($/lb) | 0.4 |
| Slope angle/levels | 45° / 5 |
| Cut-off | Profit > 0 |
| Confidence level | 95% |



# Final pit results

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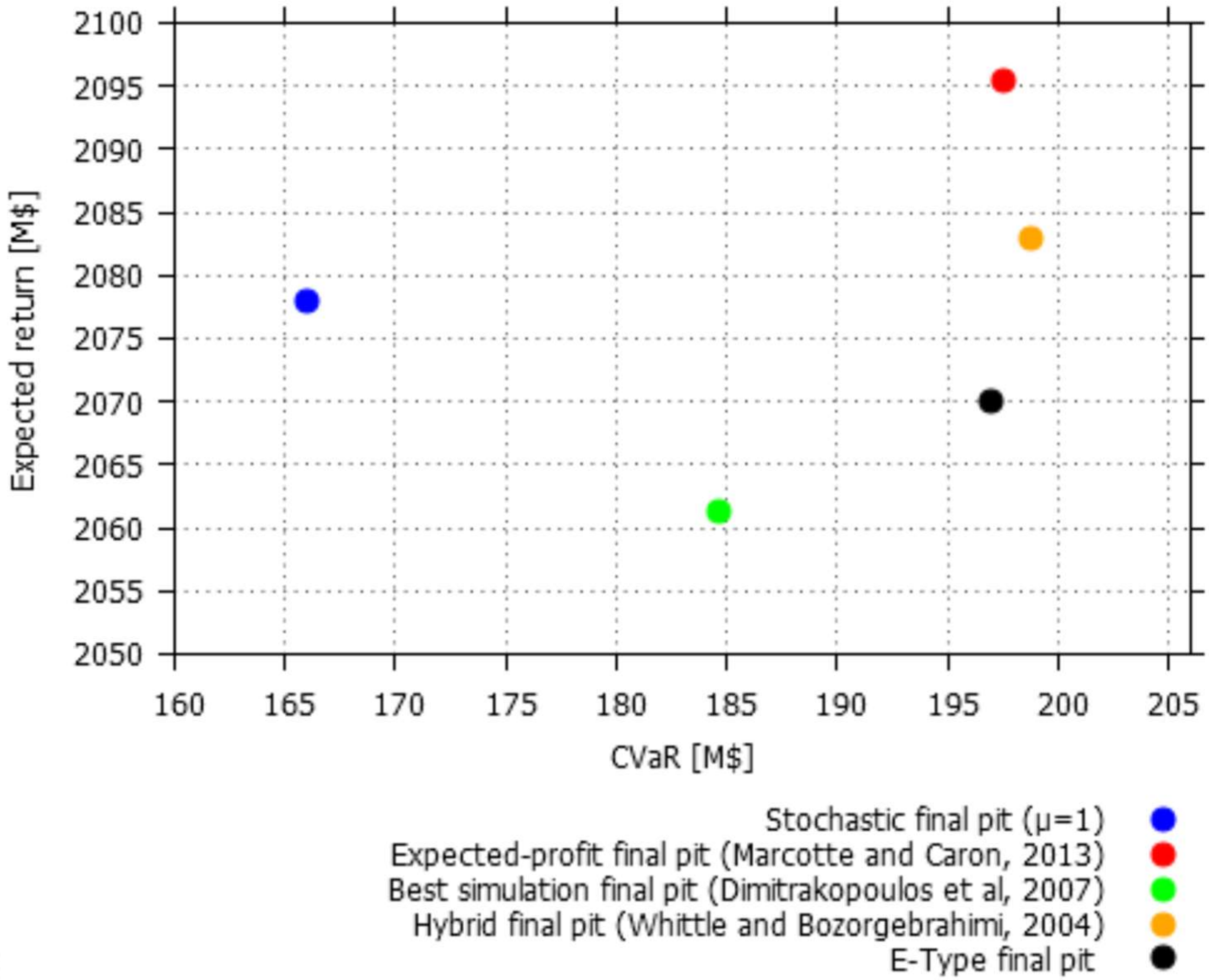
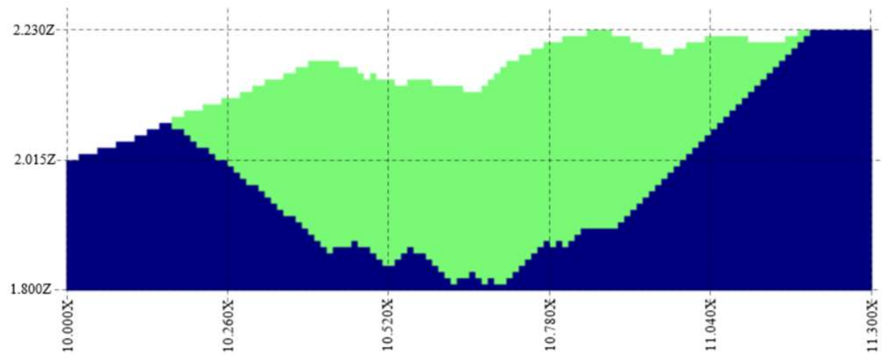
# Final pit results



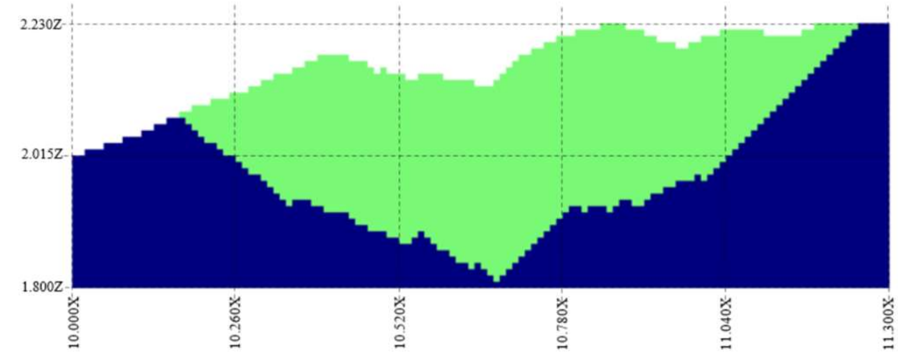

**Comparison**

- **Differences in terms of Exp Val are < ±1%, but…**
- **The risk is reduced in average ~18%**

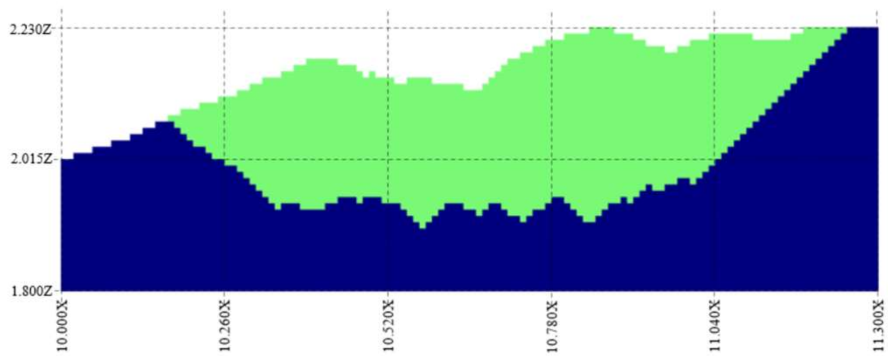
# Final pits results



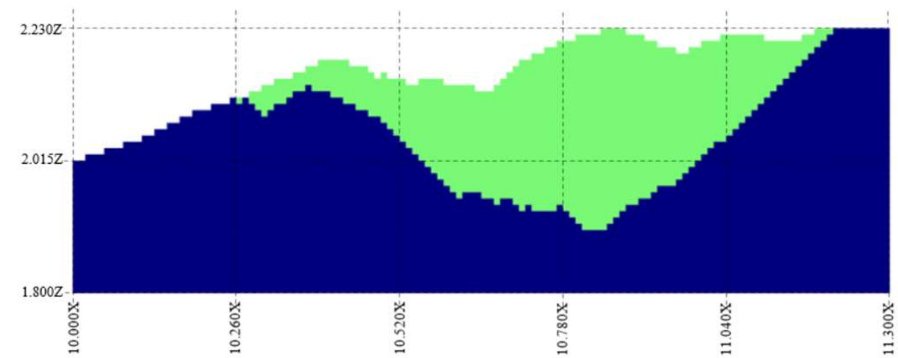

Final pit, μ=1




Final pit, μ=4




Final pit, μ=8




Final pit, μ=10

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# Conclusions

- Advantages when incorporating geological uncertainty in stochastic-risk approach
  - ☺ More expected value (a little bit,…)
  - ☺ Lower risk of loss
  - ☺ Efficient frontier: policy to manage the profit vs risk relationship

# Future work

- Improve the model:
  - Another geological characteristics
  - Study the impact on number of realizations
  - How to ensure a well-distributed generation of points in efficient frontier?

# An efficient frontier analysis to determine final pit under geological uncertainty

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