Sampling error and its effect on grade control profit[1]

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Abstract

Mineral grade prediction is a critical phase in mineral exploration and resource estimation, and it plays a vital role in the economic evaluation of mining projects. In mining, grade control is the process of identifying where the mined material will end up. Misclassification of ore grades costs money, hence the requirement for representative sample methodologies in open-pit mining is becoming increasingly critical in all mining industries. The impact of Ordinary Kriging estimation is discussed in this study. Ordinary kriging is regarded as a highly dependable method and is commonly used for estimation. A mine's blasthole data is examined. Because errors are likely to occur during grade control or even actual mining operations, data analysis is performed by introducing errors in the data. An economic analysis is performed on the various errors introduced and how they will affect the mine's profit.

1. Introduction

Mining companies want to increase the returns on the investment made in the mining operation, hence the need to optimize grade control to minimize misclassification of ore and waste. Grade control is a technique that offers selectivity for the extraction of different types of ore and waste with the aim of increasing profit or minimizing loss in the mining operation (Verly, 2005). According to the characteristics of the ore, the detailed data gathered at the grade control stage is used to separate ore from waste, a process known as "ore-waste classification," and to determine the final destination of the various material types (Abzalov et al., 2010).

The quality and quantity of the samples used determine the effectiveness of ore grade control at active mines. The simplest implementation of grade control consists of manually designing ore-waste boundaries or ore blocks on a map of blast hole grade values (Verly, 2005). In grade control, data can be viewed as a distribution, normally as a histogram, or as spatial continuity, where data is analyzed as a variogram. The main argument for using simulations is that smoothed maps obtained by kriging do not account for the uncertainty in the grade estimation or for the economic consequence of misclassification. As a result, optimum classification cannot be achieved (Verly, 2005). Geostatistical simulation methods for grade control have been used to solve optimization problems such as minimizing loss functions or maximizing the expected profits (Glacken, 1997; Deutsch, Magri and Norrena, 1999).

This paper aims to compare the estimated grade using the Nearest Neighbor, Inverse Distance Weighted, Inverse Distance Weighted Squared, and Ordinary Kriging to the true grade of a dense grid, which is the grade produced through simulation, and discuss its effect on profit. An open pit mine in Chile will be used

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as a case study, with the estimated grade compared to the true grade using a cutoff. GSLIB software will be used for generating variograms and simulating the blasthole data.

2. Literature review

Grade control procedures depend on both the quality and quantity of samples. Just improving the sample quality does not always lead to better defined ore and waste blocks if the spacing chosen is too broad. The primary goal of grade control in mines is to distinguish between material that is above cut-off grade and material that is below cut-off grade by estimating recoverable reserves. Because drilling does not cover the entire area to be mined, recoverable resource estimation attempts to predict the quality (grade) and quantity (tonnage) from a limited number of data points (Gulule, E. P., 2016).

According to an estimation of economic losses due to poor blast hole sampling in open pits, errors in sampling and preparation as well as estimation methodology are to blame for losses of the order of millions of dollars annually. The estimation methodology is responsible for larger losses than those related to sampling errors. The use of the polygonal method instead of kriging generates invisible profit losses. Kriging is less sensitive to the level of sampling error than the polygonal method. Other techniques, such as geostatistical simulations, could be evaluated to improve the SMU classification. The total profits can be significantly impacted by improvements like the adoption of geostatistical estimation methods, better equipment for sampling and sample preparation, and staff training (Magri & Ortiz, 2000). Geostatistical simulation allows for the quantification of losses generated by poor blast hole sampling and imperfect estimation.

Inverse Distance Weighting methods assume that samples taken close together will have more characteristics in common than samples taken farther apart, with anisotropies not frequently considered when using the same corner point grade and thickness for multiple orebodies. Kriging is superior to the IDW method because it considers not only distance but also spatial variability, as well as sample redundancy and proximity. Outlier values in geostatistics can result in distorted variograms due to the high nugget effect. The main aim of cutting the high-grade values is to alter the samples' distribution.

Kriging uses a set of simultaneous linear equations for each point on the output grid such that all the actual input data is optimally weighted according to distance using the semi variogram. Kriging is the geostatistical estimation method developed to provide the optimal linear and unbiased estimates. It depends on expressing spatial variation of the property in terms of the variogram (or correlogram), and it minimizes the prediction errors, which are then estimated. The technique is based on the assumption that the variable to be estimated is a regionalized variable. The technique is used if the underlying conditions of second-order stationarity are met, which means that the sample data's mean and variance stay unchanged in space at a minimum. Ordinary kriging is the most widely used kriging method. It serves to estimate a value at a point of a region for which a variogram is known, using data in the neighborhood of the estimation location. Ordinary kriging can also be used to estimate a block value (Wackernagel H., 1995).

3. Case study

3.1. Background

The case study's data came from a copper mine in Chile, South America. This dataset contains 28,634 blasthole data points collected from a 15 m bench height open pit. The copper grade reported has a mean, standard deviation, and coefficient of variation of 0.841, 0.330, and 0.392, as shown in the histogram in Figure 1.

Sorting the data by elevation resulted in the selection of a bench. The bench data chosen ranged between 270 and 285 meters. On the chosen bench, 1855 blasthole data were discovered. According to the histogram in Figure 2, the mean, standard deviation, and coefficient of variation of the copper grade reported are 0.815, 0.203, and 0.249, respectively. Figure 3 shows the location of the blast hole as well.

The following GSLIB functions will be used to simulate blasthole data: **nscore**, **vmodel**, **sgsim**, and **blkavg**. **Histplt** and **pixelplt**, on the other hand, will be used to visualize the output files.

The normal score values for the numerous blast holes are calculated. The normal score feature alters the dataset to closely resemble a standard normal distribution. This is performed by comparing the ranks provided by a normal distribution to the ranks acquired by ordering the values in the dataset from lowest to highest. When the word "normal score" is used, it is typically expected that the result can be compared to a table of standard normal probability. Figure 4 depicts the normal score values. A typical 8-directional variogram with 40 lags, 10m lag separation distance, and 5m lag tolerance is generated from the normal score output file, as shown in Figure 5.

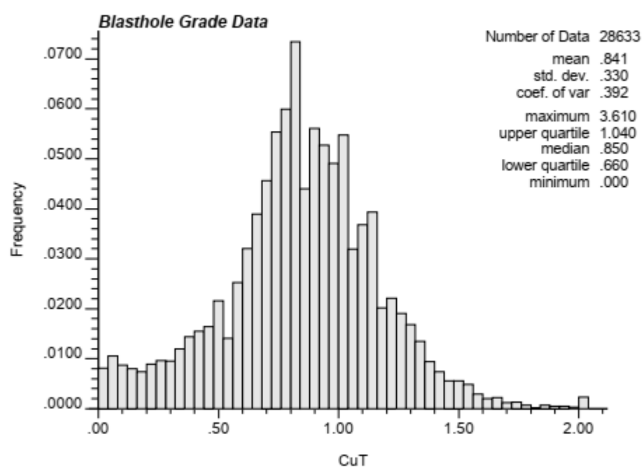

Figure 1: Histogram of the grade of all the blast hole data obtained from the mine.

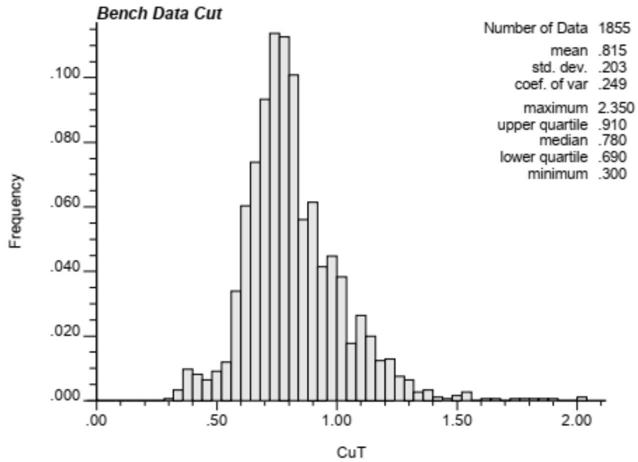

Figure 2: histogram of the grade of the selected blast hole data.

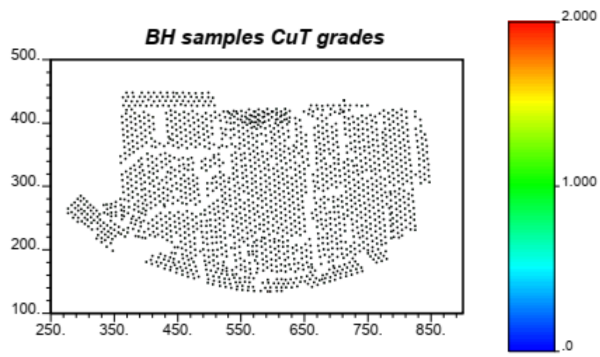

Figure 3: map of blast hole samples for Cu grade.

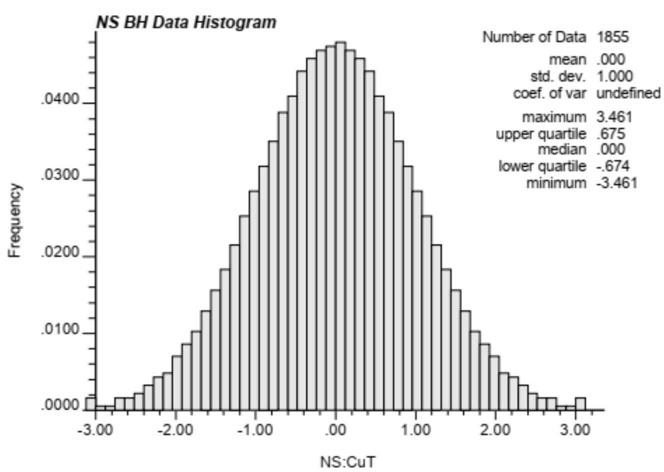

Figure 4: Histogram showing the normal score of the selected data.

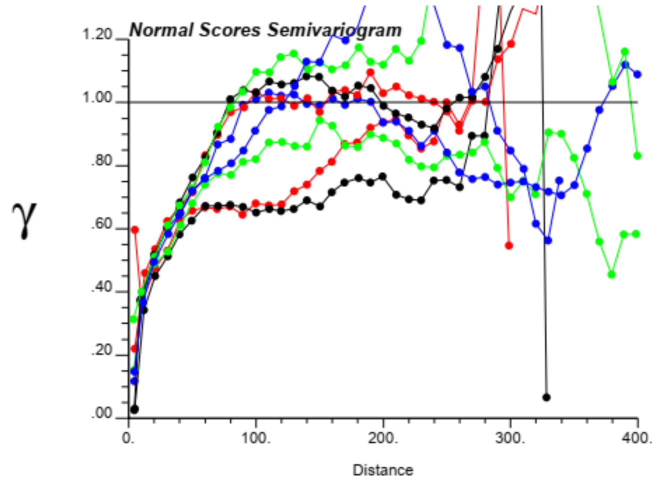

Figure 5: normal score directional variograms.

The second and sixth azimuth directions of 22.5 and 112.5 fall in the outer section of the normal score semi variogram shown in Figure 5. A variogram model is then created for the two dimensional variogram. Figure 6 depicts this.

The selected blasthole data is then utilized to produce a simulated model using sequential Gaussian simulation with the corresponding variogram model. SGSIM, a stochastic method, was developed to avoid the smoothing effect caused by deterministic methods by generating a number of stochastic realizations. Actual data from sampled sites and values from previously simulated locations are used in the sequential procedure to inform each unknown location. Furthermore, because distinct random paths are constructed that can pass through the unsampled sites in different orders, SGSIM can generate a number of equally plausible outcomes to investigate and evaluate the uncertainty (Verly, 1993). The simulation was implemented with a maximum search radius of 50m in all directions. A single simulation with a minimum and maximum original data of 8 and 12, respectively, is used as a representation of the ground truth distribution of grades at point support. Figure 7 depicts this simulated scenario.

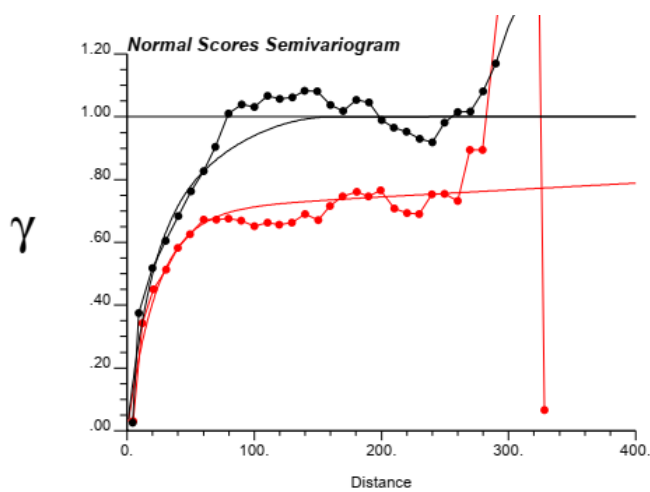

Figure 6: Normal Score variogram model.

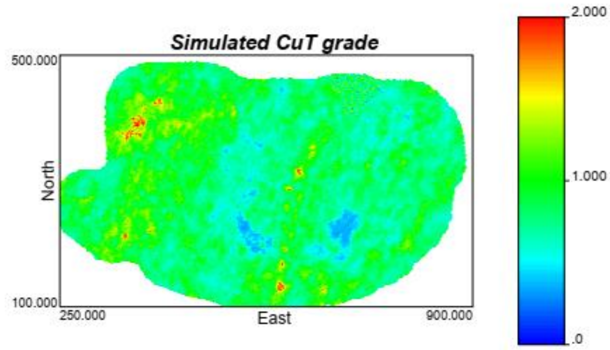

Figure 7: Simulated point Cu grades.

The ground truth at point support is used to generate the block ground truth, by averaging the simulated points inside each 10 x 10m block. The Ground Truth Cut block grades are shown in Figure 8.

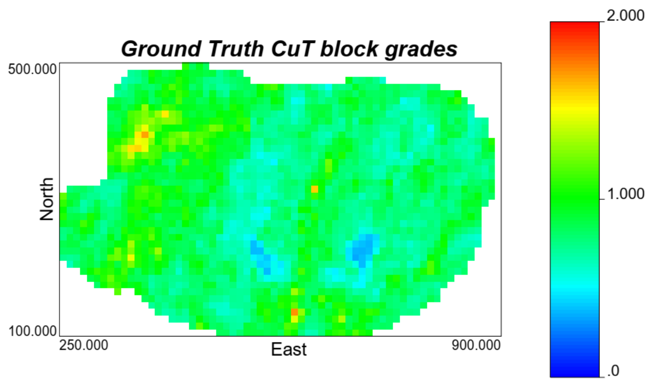

Figure 8: Simulated block Cu grades.

3.2. Addition of errors and block estimation

Ordinary Kriging is preferred in block estimation because, despite the assumption that the mean is unknown, it assumes stationarity on the neighborhood of the estimate point. Estimation is performed over 10 x 10m blocks and using a 4 x 4 block discretization, with minimum and maximum data for kriging of 4 and 16, respectively. Ordinary kriging is conducted using the previously developed variogram model. Figure 9 depicts the estimated block after using Ordinary Kriging.

Errors are introduced in the copper grade recorded in the blasthole data. These errors are used to assess the effect of precision of the data as well as the consequences of having biased data. A random error is added to the blasthole data to evaluate precision when 10%, 20%, 30%, and 50% errors are introduced. For the bias scenarios, copper grades of 110%, 90%, 130%, 70%, 150%, and 50% of the true blasthole sample value are calculated.

Ordinary kriging is then used to estimate the block grades. Negative grades that appear during the estimation are made zero. The estimated block models resulting from using samples with precision errors are shown in Figure 10, while those resulting from using samples with bias are seen in Figure 11, showing the different scenarios. A statistical summary of the various errors is shown in Table 1 and Table 2.

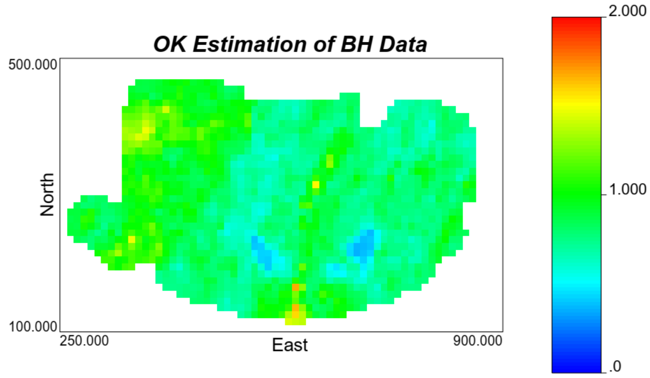

Figure 9: Ordinary kriging estimation of Cu grades at block support.

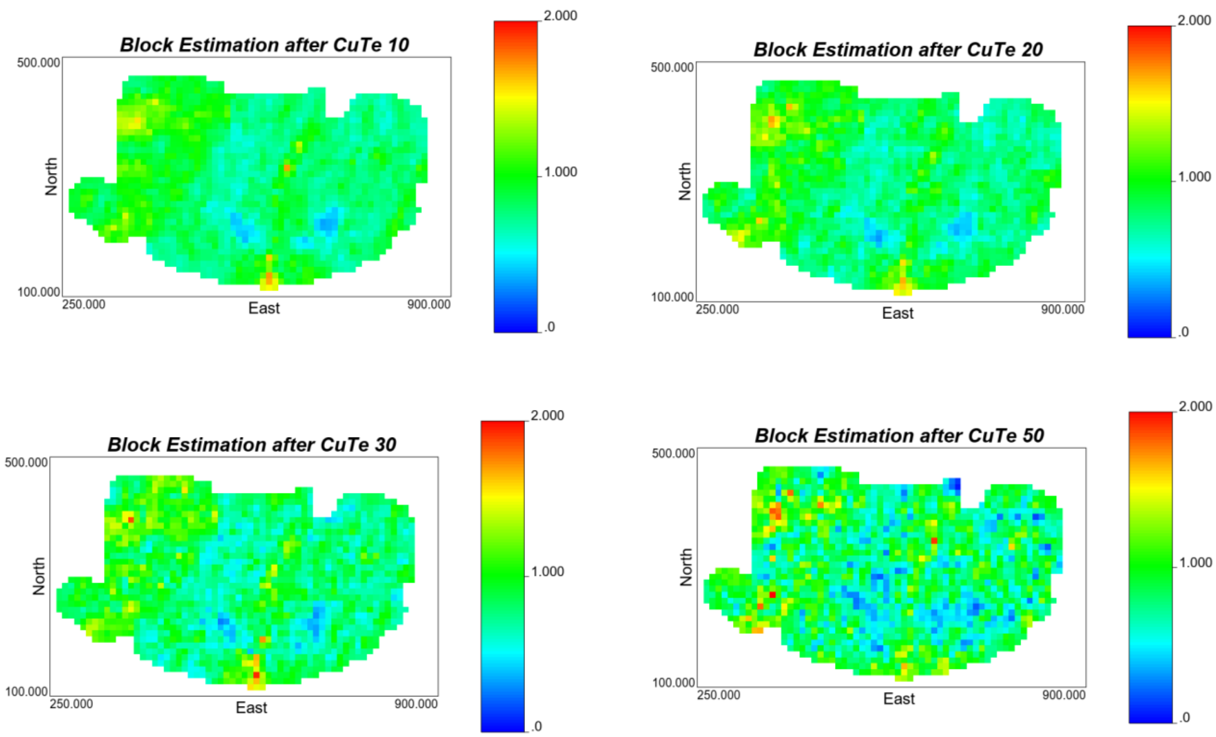

Figure 10: Block estimates based on samples with added precision errors of 10% (top left), 20% (top right), 30% (bottom left) and 50% (bottom right).

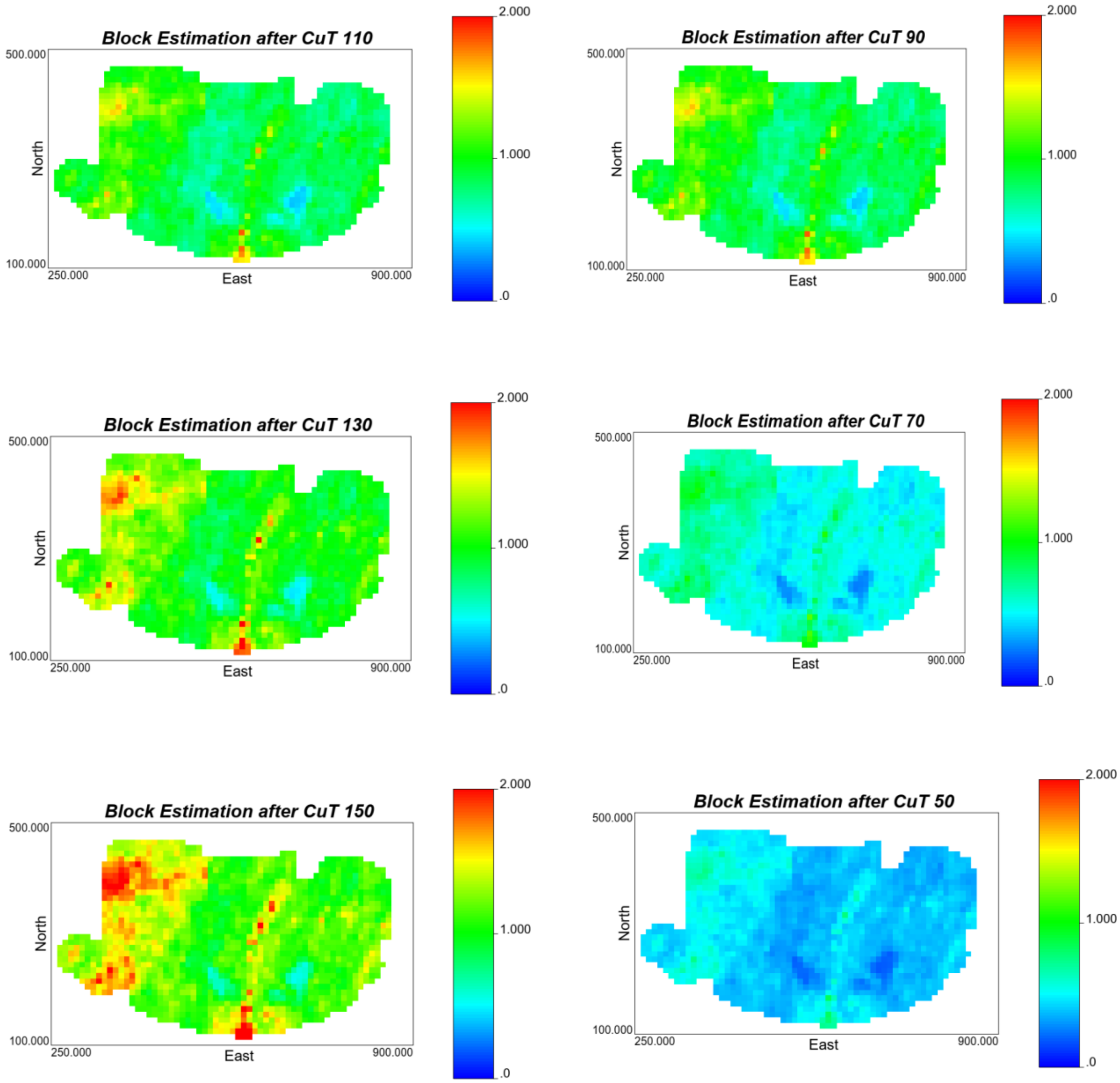

Figure 11: Block estimates based on samples with added bias of +10%, +30% and +50% (left column), and -10%, -30% and -50% (right column).

Table 1: statistical summary of ground truth point and block simulated values, and original blasthole samples and samples with added error for different precisions.

Parameters	SIM point	SIM block	BH data	Prec. 10%	Prec. 20%	Prec. 30%	Prec. 50%
number of data	49005	2037	1855	1855	1855	1855	1855
mean	0.820	0.820	0.815	0.817	0.820	0.811	0.795
standard deviation	0.188	0.165	0.203	0.220	0.265	0.323	0.454
coefficient of variation	0.229	0.202	0.249	0.269	0.323	0.399	0.560
maximum	2.350	1.711	2.350	2.207	2.282	3.260	2.851
upper quartile	0.910	0.904	0.910	0.924	0.962	0.988	1.066
median	0.790	0.797	0.780	0.782	0.787	0.775	0.760
lower quartile	0.700	0.714	0.690	0.675	0.640	0.594	0.477
minimum	0.181	0.329	0.300	0.281	0.219	0.000	0.000

Table 2: statistical summary of blasthole samples with bias.

Parameters	SIM point	SIM block	Bias +10%	Bias -10%	Bias +30%	Bias -30%	Bias +50%	Bias -50%
number of data	49005	2037	1855	1855	1855	1855	1855	1855
mean	0.820	0.820	0.897	0.734	1.060	0.571	1.223	0.408
standard deviation	0.188	0.165	0.223	0.182	0.263	0.142	0.304	0.101
coefficient of variation	0.229	0.202	0.249	0.249	0.249	0.249	0.249	0.249
maximum	2.350	1.711	2.585	2.115	3.055	1.645	3.525	1.175
upper quartile	0.910	0.904	1.001	0.819	1.183	0.637	1.365	0.455
median	0.790	0.797	0.858	0.702	1.014	0.546	1.170	0.390
lower quartile	0.700	0.714	0.759	0.621	0.897	0.483	1.035	0.345
minimum	0.181	0.329	0.330	0.270	0.390	0.210	0.450	0.150

3.3. Cutoff grade application

In analyzing the effect of the errors introduced, a cutoff is applied. To avoid issues related to mine and processing capacity, the cutoff selected is close to the breakeven cutoff grade, in this case 0.554% Cu. The ore blocks estimated grades are used to decide to send the block to the processing plant, while the actual grade (the one from the block support simulated ground truth) is used to assess its value. Since block grade estimates are based on samples and these can have sampling errors related to precision or bias, the ore/waste classification will be imperfect, which has an impact on the overall economic value.

In calculating the tonnage of the blocks a bulk density of 2.7 t/m^3 is assumed. The height of block is 15 m with the block area of 10m x 10m. The metal content for all the scenarios is derived from the multiplication of the grade and tonnage of the block and a recovery of 90% is assumed.

3.4. Economic evaluation of the blocks

This computation determines the profit or loss of each block and bench under discussion, taking into account the costs to be incurred and compensations to be made for the cutoff grade used.

The blocks with an average grade higher than the cutoff grade are termed ore, which is computed by deducting the cost of processing from the value of the block in terms of its grade. Waste blocks give no economic advantage. These results are added together to determine how much profit or loss will be earned in the sequence of scenarios mentioned to check for precisions and biases. Notice that both ore and waste blocks need to be removed from the bench, so the mining cost is not considered in these calculations.

The values of all parameters used are based on assumptions about a real-world scenario. The parameters used in the evaluation are shown in Table 3. The grade assigned to each estimated and error-added block is based on the simulated grade, since this constitutes the true grade of the block and determines the profit or loss actually incurred by the operation. The decision about where to send the block, on the other hand, is made on the estimated grade, which may have been obtained from samples with error. If the estimated grade of the block is greater than the cutoff grade, the block becomes an ore block, with its economic evaluation based on the simulated grade of the block. However, if the grade of block estimated is less than the cutoff grade, the block becomes a waste block, for which the economic evaluation is the cost of mining the entire block. Each block was estimated to be 10m x 10m x 15m in size. A bulk density of 2.7 was used in determining the tonnage of the block, which is denoted by the block volume multiplied by the bulk density. The outcome of the evaluation is shown in Table 4 and Table 5 for the precision errors and biases. A graphical representation of the outcome of the evaluation is shown in Figure 12 and Figure 13.

Table 3: economic parameters used in the profit calculations.

Parameters	Value	Units
Price	3.3	USD/lb
Recovery	90	%
Processing cost	22	USD/t
Ton	10x10x15	Ton/block
Metallurgical cost	1.3	USD/lb

Table 4: ore blocks, their grade, tonnage, metal and profit, along with their relative error with respect to the ground truth, for different scenarios of blasthole samples precision errors.

Cutoff 0.554%Cu Parameter	SIM block	BH	Prec. 10%	Prec. 20%	Prec. 30%	Prec. 50%
Ore Blocks	1526	1538	1531	1520	1448	1297
Average grade	0.83	0.83	0.83	0.83	0.84	0.84
Tonnage (kT)	6180	6229	6201	6156	5864	5253
Metal (M lb Cu)	113	114	114	113	108	98
Profit (M$)	68.0	68.0	67.9	67.6	66.0	60.1
Grade error (%)	**0.0**	**-0.3**	**-0.2**	**-0.1**	**0.8**	**1.3**
Tonnage error (%)	**0.0**	**0.8**	**0.3**	**-0.4**	**-5.1**	**-15.0**
Metal error (%)	**0.0**	**0.5**	**0.2**	**-0.5**	**-4.4**	**-13.9**
Profit error (%)	**0.0**	**-0.1**	**-0.2**	**-0.6**	**-3.0**	**-11.6**

Table 5: ore blocks, their grade, tonnage, metal and profit, along with their relative error with respect to the ground truth, for different scenarios of blasthole samples bias errors.

Cutoff 0.554%Cu Parameter	SIM block	BH	Bias +10%	Bias -10%	Bias +30%	Bias -30%	Bias +50%	Bias -50%
Ore Blocks	1526	1538	1548	1488	1562	811	1576	1526
Average grade	0.83	0.83	0.83	0.84	0.82	0.94	0.82	1.20
Tonnage (kT)	6180	6229	6269	6026	6326	3285	6383	421
Metal (M lb Cu)	113	114	114	111	115	68	115	11
Profit (M$)	68.0	68.0	67.9	67.8	67.7	50.1	67.4	10.8
Grade error (%)	**0.0**	**-0.3**	**-0.5**	**0.7**	**-0.9**	**12.9**	**-1.4**	**44.2**
Tonnage error (%)	**0.0**	**0.8**	**1.4**	**-2.5**	**2.4**	**-46.9**	**3.3**	**-93.2**
Metal error (%)	**0.0**	**0.5**	**0.9**	**-1.8**	**1.4**	**-40.0**	**1.8**	**-90.2**
Profit error (%)	**0.0**	**-0.1**	**-0.2**	**-0.4**	**-0.5**	**-26.3**	**-1.0**	**-84.2**

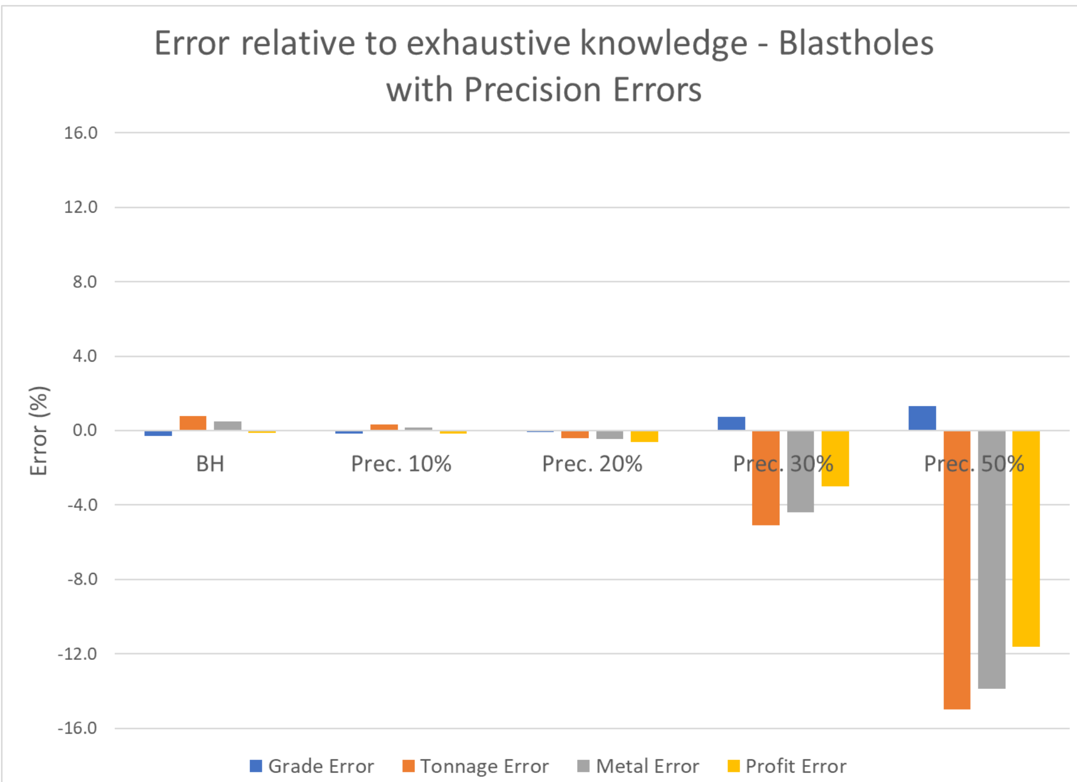

Figure 12: relative errors in grade, tonnage, metal and profit for the case of blasthole samples with precision errors.

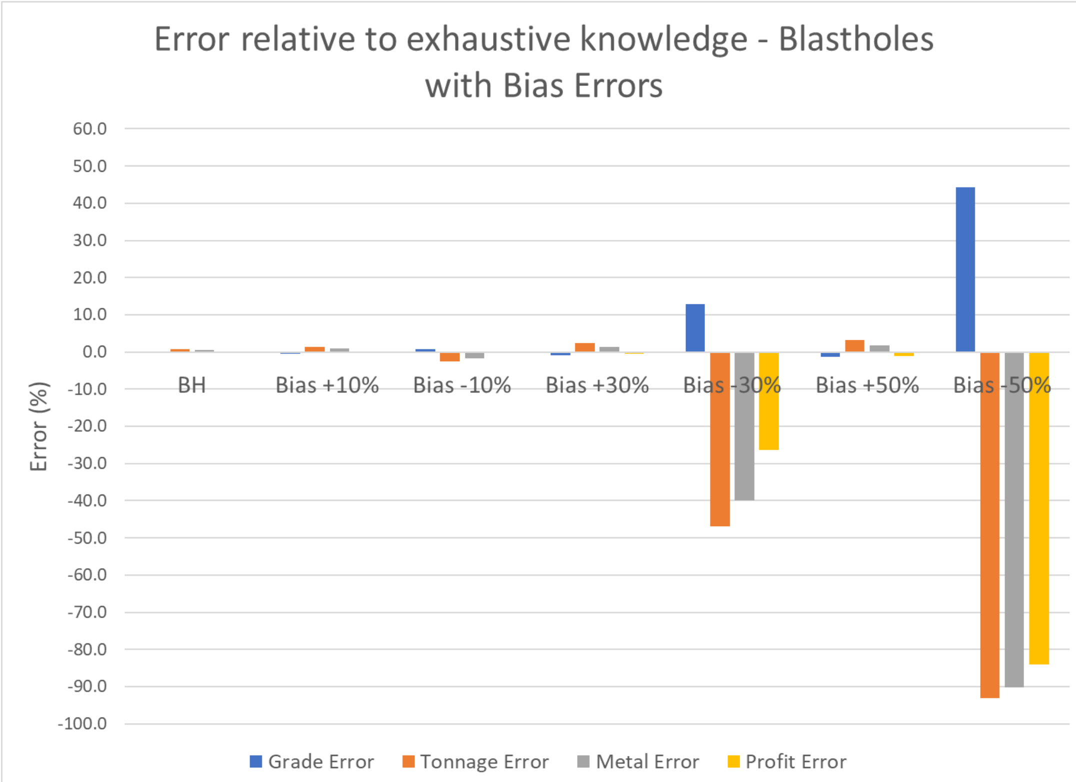

Figure 13: relative errors in grade, tonnage, metal and profit for the case of blasthole samples with bias errors.

3.5. Results discussion

The results demonstrate that the use of samples with errors lead to a short term block model that misclassifies the correct destination of ore and waste blocks. This translates into a loss of profit.

The use of unbiased and precise samples at blastholes, along with an unbiased and optimum estimation technique such as ordinary kriging, generates a model that is very close to the perfect classification of blocks, which in practice is unachievable, since we do not have access to the true block grades. In this work, that "reality" is based on one simulated model, developed at point support and then block averaged to represent the true grade of blocks. The points simulated are conditioned to the actual blasthole dataset of the mine. The classification using the blasthole samples without any added error, generates relative errors in the ore grade, tonnage, metal content and profit lower than 1%. Profit is only 0.1% lower than the unachievable case of perfect knowledge. This is encouraging and should suggest to practitioners that ordinary kriging must be used in short term planning and during grade control to determine the block grades prior to their assignment as ore or waste.

When samples suffer from precision errors, their values are overall unbiased, but at any location they can be slightly higher or lower than the actual value. Depending on the level of noise, the estimation of the block grades will be affected proportionally, and this means more blocks are incorrectly estimated above or below the cutoff. Blasthole sampling is known to be of poor quality in many mining operations. However, quality control and quality assurance procedures ensure that it will rarely exceed 15 to 20% of relative error (this is the error relative to the mean value of the sample grades). It can be seen that when the error in precision is 10 or 20% in the blasthole samples, the estimated blocks will be misclassified more often. However, the relative errors in grade, tonnage, metal content and profit are still below 1% with respect to the case of perfect knowledge. Profits decrease in 0.2 and 0.6% for 10% and 20% precision errors. However, when the samples precision is lower, with errors of 30 and 50%, the impacts on profit explode quickly. Profit falls by 3.0 and 11.6% for these two cases, which represents a loss of $2 million and $8 million, respectively. This is a loss over just over 6 million tons of ore. In a large open pit, this could represent a month of production, that is, $2 million or $8 million per month.

When samples are biased, losses can be very significant, depending on the cutoff and the direction of the bias. A positive bias will send more material to the mill, which in the end may not generate too much of a loss, if the "waste" is not too low in its true grade. On the other hand, if the bias is negative, many blocks of ore will be sent to the waste dump, since their grade was predicted with a systematically lower value. The few blocks that are correctly assigned to the processing plant will not generate enough metal to compensate for the lost ore. We see significant profit losses for biases of -30% and -50%. Losses in other cases are minor, although they are always there.

4. Conclusions

The study shows that ordinary kriging is a great tool for forecasting the true block grades and the profit losses due to the lack of perfect knowledge of the true grades is very small. When samples are affected by sampling errors, the effects can be tremendous in profit. Precision issues may lead to million dollars losses over a year's production. Furthermore, when samples are biased, this may lead to a loss of ore, which will be misclassified and sent to the waste dump. Comparison with other estimation methods would provide insight with respect to their unbiasedness, optimality and robustness. Also, accounting for mining and processing capacity constraints would change the results. These possibilities could be explored in future work.

5. References

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