A review of grade control methods in open cast mining[1]

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Abstract

Ore grade control is an important part of a mine's short-term planning to define which material is considered as ore or waste. The effectiveness of grade control is subject to a compendium of factors such as sampling errors, conditional bias introduced by grade estimators, accurate definition of dig limits, and blast-induced rock movements. In this scenario, misclassification of the mined material which is mainly ascribable to the lack of absolute knowledge about real grade distribution, is our major concern. This paper reviews the state-of-the-art grade control practices used for classifying ore and waste in open pit mining. Common approaches include the classical and distance weighting estimation techniques, geostatistical methods such kriging, and simulation-based methods. Theoretical review shows that conditional simulation is a better classifier of ore and waste than estimation methods due to its ability to account for grade uncertainties and the different optimization algorithms it provides to access economic consequences of ore/waste decisions. Subsequently, a novel machine learning approach using Elliptical Radial Basis Function Network (ERBFN) and Support Vector Regression (SVR) was discussed. Results from a case study show that SVR achieved an 8%, 1.12% and 1.16% reduction in misclassified material relative to inverse distance, ordinary kriging, and simulation respectively. The ERBFN model also obtained a decrease in misclassified material of 12%, 5.4% and 5.7% compared to inverse distance, ordinary kriging, and simulation-based approaches, respectively.

1. Introduction

Considering the extensive nature of mining operations which involves making decisions regarding large volumes of materials taking place over limited period of times, blending of waste with ore and ore with waste is inevitable. Nevertheless, geologists and mine engineers must ensure that this situation is reduced to the barest minimum during excavation, and that ore and waste are differentiated. The decision of which material is ore or waste is very crucial for the mine's profitability since it is the last opportunity for the mining company to achieve its estimated revenue, and errors at this stage are very costly and difficult to reverse due to its proximity to the production stage (Rossi & Deutsch, 2014). To help make better decisions in classifying ore and waste, and to select the destination of each parcel of material mined, mines perform grade control.

Ore grade control is a compendium of procedures and practices usually involving blast hole sampling, grade estimation, ore/waste classification, blast-induced rock movement measurements, defining dig limits among others. It is done to identify which material is ore or waste and to ensure that the mill is fed

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with the right grade of material. In this scenario, material classification based on the grade assignment is discussed. A volume of material is classified as ore or waste based on the grade assigned to it from grade prediction methods and by comparison to a cut-off grade (Abzalov, 2016). When the grade assigned to a material falls below a given cut-off grade, it is considered as waste and sent to the waste dump while material with grade above the cut-off grade is sent to the mill. However, when the assigned grades are not consistent with the actual grade distribution, it results in a misclassification, and materials are sent to the wrong destination. The figure below shows the basic issue of misclassification where a scatterplot of true grades for each block are plotted against the corresponding predicted grades.

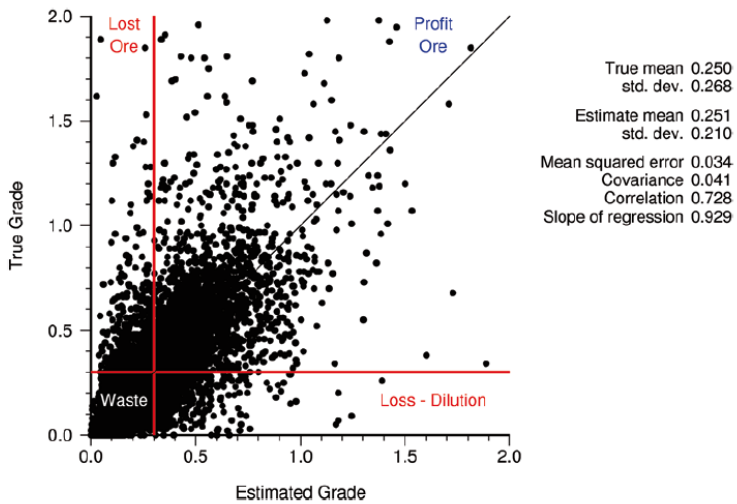

Figure 1: Misclassification in grade control. Source: (Rossi & Deutsch, 2014).

Common approaches include the classical nearest neighbour method where grades of the closest blasthole sample are assigned directly to a block model, and the inverse distance weighting estimation method which assign grades by calculating the weight of each sample that is inversely proportional to the distance of the estimation location (Ortiz, 2020). Geostatistical methods such as kriging were introduced in the early 1950s. Kriging minimizes the estimation variance under certain conditions (Rossi & Deutsch, 2014). Conditional simulation methods have been the most advocated technique in recent years to predict grades and their uncertainty, thus addressing some of the shortcomings of kriging and the classical methods. This method simulates grades at given locations and can be combined with different optimization algorithms where the material types (ore or waste) are evaluated against all simulated realizations and the optimum destination or material type at each location determined (Vasylchuk & Deutsch, 2018).

In this paper (Vasylchuk & Deutsch, 2018), the authors review four grade interpolation methods:

1. Nearest neighbour polygonal method;
2. Inverse distance weighting method;
3. Ordinary kriging; and
4. Simulation-based methods.

The paper also discusses the performance of two novel machine learning (ML) algorithms that have been used recently in grade control: Elliptical Radial Basis Function Network (ERBFN) and Support Vector Regression (SVR). The results from these methods are analysed with the help of the case studies presented to show which method is most efficient for grade control.

In the next sections, grade control methods are reviewed and critically assessed in a simple theoretical framework. Results obtained in the literature from numerical experiments conducted to compare the effectiveness of simulation versus different estimation methods, as well as results on a comparative case study at the Carmen de Andacollo copper mine (Chile) are presented. Summary and conclusions follow.

2. Methods for grade prediction

Grade prediction is probably the most crucial aspect of grade control because it forms the basis for selecting ore and waste zones. The main purpose of estimation is to predict the grade of the variable at unsampled locations in the block model which is the premise for classification. Samples taken from blast holes are analyzed from which a quantitative model of the ore body is constructed by interpolating and extrapolating between these samples to account for the grade in areas that were not sampled. This is with the assumption that all the sample locations and the unsampled location belong to the same domain. There are various methods developed for performing grade prediction in grade control, however this paper discusses the most common ones in the industry.

2.1. Nearest Neighbour (NN) estimation method

The nearest neighbor method is one of the simplest approaches to grade estimation. As a variant of the polygonal method, the nearest neighbor assigns the grade of the closest blast hole sample to the entire unsampled block (Vasylchuk, 2019). Since the weight of each sample is an important factor in estimating, NN method determines the weight of the samples by assigning all the weight to the closest sample and every other sample gets a weight of zero. In relation to the cut-off grade, the NN method regards the estimation location as ore if the grade of the closest sample is larger than the cut-off grade. The value at the unsampled location is then calculated as shown in the equation below:

$$z_{NN}^*(u_o) = \lambda_o^{NN} + \sum_{i=1}^{n} \lambda_i^{NN} z(u_i) \tag{1}$$

$$\lambda_i^{NN} = \begin{Bmatrix} 1 & if\ u_i\ is\ closest\ to\ u_o \\ 0 & otherwise \end{Bmatrix} \qquad i = 1, \dots, n$$

$$\lambda_0^{NN} = 0$$

where $z_{NN}^*(U_o)$ is the nearest neighbor estimate at the unsampled location (u_o), λ^{NN} are the nearest neighbor weights of the samples, and $z(u_i)$ is the known grade of the samples for which $i = 1, \dots, n$ are the total number of samples.

One main strength of the nearest neighbour method is that it does not smooth estimated values (Kapageridis, 2014). However, study has shown that this method is not particularly accurate and cannot be trusted because it neither takes into account the spatial continuity of the grade nor the redundancy in the information (Ortiz, 2020). The discrepancies in this method are known to be larger that other estimators, and for many deposits that have positively skewed distributions, significant errors in the estimate occur in the individual blocks leading to proclivity to overestimate the average grade and underestimate tonnage above cut-off (Rossi & Deutsch, 2014). Nonetheless the nearest neighbour method can be used as a checking tool.

2.2. Inverse Distance Weighting (IDW) estimation method

The inverse distance weighting technique is an enhancement of the classical polygonal method, and is most suitable for uniform orebodies (Abuntori et al., 2021). This method is used to estimate grade values using several nearby blasthole sample grades to obtain a weighted average for each block as shown in Figure 2. The calculation of the estimate is shown below:

$$z_{IDW}^*(u_o) = \lambda_0^{IDW} + \sum_{i=1}^{n} \lambda_i^{IDW} z(u_i) \qquad i = 1, \dots, n \tag{2}$$

where $z_{IDW}^*(u_o)$ is the inverse distance estimate at the unsampled location (u_o), λ^{IDW} is the inverse distance weight assigned to each known sample and $z(u_i)$ is the grade of each known sample for which $i = 0, \dots, n$ are the total number of samples. In this scenario, the weights assigned to each sample are inversely proportional to the distance from estimation location and are calculated as shown in equation 3 below. Each sample is weighted based on its proximity to the location to be estimated.

$$\lambda_i^{IDW} = \frac{1/(c+d_{io}^w)}{\sum_{i=1}^{n} 1/(c+d_{io}^w)} \qquad i = 1, \dots, n \tag{3}$$

$$\lambda_0^{IDW} = 0$$

for which λ_i^{IDW} is the inverse distance weight of sample i, d_{io} is the distance between the estimation location and sample i, w is the inverse distance weighting power and c is a small constant for numerical stability or computational reasons. When the weighting power approaches zero, the weights become similar and is calculated as the arithmetic average of the samples (Abzalov, 2016). On the other hand, a larger weighting power assigns all the weight to the closest sample making the inverse distance weight similar to the result of the polygonal nearest neighbour method (Ortiz, 2020). In practice, the most frequently used weighting power is 2, however powers of 1 and 3 are also used for estimation. Even though the inverse distance weighting method provides better estimates than the nearest neighbor method, it does not account for the details of the data configuration or the varying anisotropy at different scales (Vasylchuk, 2019).

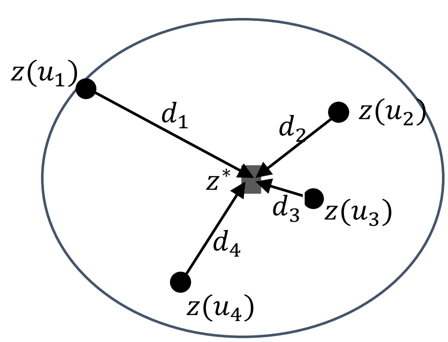

Figure 2: Inverse Distance Weighting method.

2.3. Kriging-based estimation method

Geostatistics, developed in the early 1950s, forms the basis for kriging (Krige, 1951). The geostatistical concept provides the platform for describing and modelling the spatial continuities of the regionalised variables (in this case the grade values) and allows incorporation of the continuity factors into the regression techniques used for the spatial predictions (Abzalov, 2016). Kriging is a collection of generalized linear regression techniques based on calculating optimal weights that minimize the expected error variance or the estimation variance (Ortiz, 2020). It produces an estimate that is a weighted linear combination of the data, minimizes the estimation error, hence, it called the Best Linear Unbiased Estimator (BLUE).

Kriging-based grade control came to light in open pit mines during the 1980s (Deutsch et al., 2000). Different types of kriging algorithms have been used in grade control, but most commonly ordinary kriging (OK) particularly in gold mines in Northern Nevada (Rossi & Deutsch, 2014). Other types of kriging have also been applied such as the indicator kriging and not too popular fuzzy kriging (González, 2012). Although not very common, simple kriging (SK) has also been used to estimate but more often used as a checking tool.

Ordinary kriging

Ordinary kriging (OK) is a robust estimator which assumes that the local mean is unknown (unlike SK which assumes a known mean), but constant within the estimation neighborhood based on the quasi second order stationarity assumption (Ortiz, 2020). To guarantee global unbiasedness, OK constrains the sum of the weights to be 1.0, and as a result the mean does not need to be known (Abuntori et al., 2021). The weights used in kriging directly depend on the choice of a variogram model for the data set. The variogram model, which is a prerequisite, enables the kriging algorithm to obtain insight on the anisotropy in the grade distribution. The ordinary kriging estimate is summarized in the equation below.

$$Ordinary\ kriging\ estimator,\ Z_{OK}^{*}(u_o) = \sum_{i=1}^{n} \lambda_i^{OK} Z(u_i) \tag{4}$$

where $Z_{OK}^{*}(u_o)$ is the ordinary kriging value at the unsampled location (u_o), λ_i^{OK} the weight assigned to each known sample (u_i), and $Z(u_i)$ is the grade of each known sample i for which $i = 1, \dots, n$ are the total number of samples. The kriging variance which measures the quality of the estimation is given by

$$Ordinary\ kriging\ variance,\ \sigma_{OK}^{2}(u_o) = \sigma_0^2 - \sum_{i=1}^{n} \lambda_i^{OK} C_{io} - \mu \tag{5}$$

where $\sigma_{OK}^{2}(u_o)$ is the kriging variance at the estimation location (u_o), σ_0^2 is the variance of the distribution, λ_i^{OK} is the weight assigned to the known sample i, C_{io} is the covariance between sample i and the estimation location (u_o), and μ is the Lagrange multiplier which is an additional parameter to help in optimality.

There are other variants of the kriging method which have proven successful in many operations such as the Breakeven Indicator Method (BEI) (Vasylchuk, 2016). The BEI grade control method is a blend of both indicator and grade kriging. It uses an ore/waste indicator variable to predict the probability of ore occurrence at a given location (Rossi & Deutsch, 2014). The indicator variable is then used to define ore or waste probability of the estimated value based on the grade of the blast holes, and the expected revenue is determined. This method was used in copper-molybdenum Ujina open-pit in Chile, together with and the classical inverse distance weighting method and the results were compared to a reference model. The BEI showed a relatively better performance than IDW and a summary of the results is shown in subsequent sections.

In summary, kriging provides good frameworks for predicting grades that are locally accurate estimates, however, the premise of estimating based on the minimization of estimation variance is not optimal for grade control (Srivastava, 1987). (Rossi & Deutsch, 2014) recounts that kriging has been only slightly more successful at grade control compared to the other classical methods because of the inherent smoothing and the inability to quantify the spatial uncertainty as shown in Figure 3.

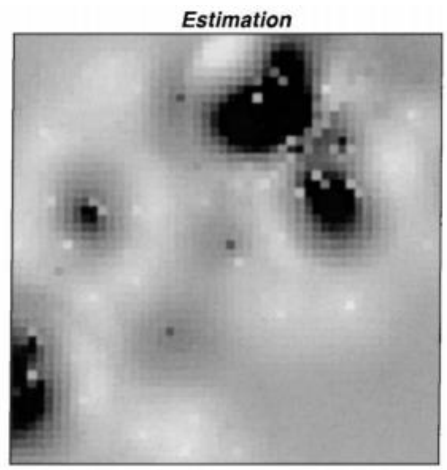

Figure 3: Smoothing effect of kriging.

2.4. Simulation-based method

The use of simulation as a predicting tool has been a cutting-edge method in grade control. The commonest simulation method used to model the realistic variability of a deposit is the Sequential Gaussian Simulation (SGS) (Vasylchuk & Deutsch, 2018). This method assigns grade to blocks and also takes into account the uncertainty in the grade distribution that can be later used for assessing economic consequences of grade control decisions, a feature lacked by the traditional estimation methods (Vasylchuk, 2019). The grade assignment process involves a series of steps such as data declustering, normal score transformation of declustered data, simulating a value from the conditional distribution and back-transforming simulated values. The simulation process returns a range of probable values from a conditional cumulative distribution function (cdf) as shown in the equation below:

$$F(u; z|(n)) = Prob\{Z(u) \leq z|(n)\} \tag{6}$$

where $F(u; z|(n))$ is the cumulative frequency distribution curve, $Z(u)$ accounts for the uncertainty in the unknown true value, and (n) represents local conditioning blast hole data within the specific neighborhood of location (u).

In order to evaluate which realization will produce the optimum classification of materials, taking into consideration its economic impact on the operation, simulation-based methods incorporate certain optimization algorithms such as minimum loss and maximum profit functions (Deutsch et al., 2000; Dimitrakopoulos & Godoy, 2014; Vasylchuk, 2016). Moreover, the economic consequences of sending ore to the waste dump is different from sending waste to the mill, hence an optimal selection criterion is needed to account for these asymmetric economic impacts through the optimization algorithms that simulation provides. The application of the economic classification functions for grade control does not only account for the penalties of each decision, but also provide essential information for non-linear metal recoveries or any other geo-metallurgical attribute of interest by adopting the economic functions (Wambeke & Benndorf, 2017).

The minimum expected loss method consists of computing the expected loss associated with each classification and selecting the classification for which the expected loss is minimal. Several mathematical expressions have been provided by different authors for the 'minimum loss' function, but for the purposes of subsequent comparison, a simplified version of the loss function by (Vasylchuk, 2016) is presented, as shown in equation 7.

$$The\ loss\ function, g(u; z, z_c) = \begin{cases} 0,\ for\ a\ correct\ decision \\ (z(u) - z_c) \times b_1, for\ an\ incorrect\ waste\ decision \\ \left(z_c - z(u)\right) \times b_2, for\ an\ incorrect\ ore\ decision \end{cases}$$

Hence, $the\ Expected\ loss\ decision = E[g(u; Z, z_c)]$ (7)

where $g(u; Z, z_c)$ represents the loss function, $E[g(u; Z, z_c)]$ is the expected loss over multiple realizations at location (u), z_c is the cut-off grade, $z(u)$ is the simulated grade value at location (u), b_1 and b_2 are the penalty coefficients for underestimating and overestimating respectively. For instance, a block will be selected as ore if the expected loss for mining the block as ore is less than the expected loss for mining it as waste.

Similarly, for maximum profit function, a block will be selected as ore if the expected profit for mining the block as ore exceeds the expected profit for mining it as waste, and vice versa. More information on concept of minimum expected loss or maximum profit as a basis for classification decisions can be found in (Rossi & Deutsch, 2014; Vasylchuk, 2016; Verly, 2005; Wambeke & Benndorf, 2017).

2.5. Machine Learning (ML) method

Technological advancements in recent years have enabled computers to process large amounts of information within the shortest possible time. Machine learning algorithms (MLA) are a collection of advanced statistical methods empowered by high-level computers to provide flexibility and simplicity when integrating and recognizing complicated patterns in data, which is a difficult task with linear geostatistical workflows. Machine learning is already incorporated to solve Earth Sciences problems (Deutsch et al., 2016) but is not fully integrated into the geometallurgical workflows that model and optimize the processes leading to the extraction and recovery of minerals and metals (Ortiz, 2019). As far as this review is concerned, the application of machine learning methods for grade assignment and classification in grade control is a novel enterprise with not too many applications. ML algorithms such as Artificial Neural Networks have been used for mineral resource estimation (Abuntori et al., 2021) but have not been explored in grade control. The application of ML is to enhance the accuracy of predicted grade values, and to make better decisions concerning the classification of mined material.

A recent work by (Da Silva et al., 2020) demonstrated the application of ensemble ML methods for grade control in the Carmen de Andacollo copper mine in Chile. Two algorithms were used in their work: Elliptical Radial Basis Function Network (ERBFN) and Support Vector Regression (SVR). The two ML algorithms are trained which was preceded by the tuning of their respective hyperparameters. For instance, the number of nodes that constitute a hidden layer in the network was defined for ERBFN. Once the nodes are defined, ERBFN algorithm trains the data and assigns a new networking system to each node to make grade predictions. The different predictions obtained are averaged to form a trend model which is further used as a secondary variable to assess the consequences of each grade control decision on an intrinsic collocated cokriging framework. Just as kriging, variogram models were generated and search parameters defined.

In the case of SVR, the data is divided into a training, validation, and test dataset. Different reference models are defined from which predictions are generated for the validation set. The predicted values and the blasthole values are then fed into a trained meta model from these pairs of values, and then final grade predictions are made. Just like ERBFN, the SVR algorithm optimizes the best scenario for ore and waste by using the final predicted model in a collocated cokriging framework. Finally, based on a break-even cut-off grade defined by the mine operations, the destination of each material is determined. Details of this work can be found in (Da Silva et al., 2020).

3. Results and Discussions

The results presented here are case studies reviewed from (Da Silva et al., 2020; Rossi & Deutsch, 2014; Vasylchuk, 2016) which demonstrate the performance of grade prediction methods. The first scenario is a study in a copper-molybdenum Ujina open pit mine in Northern Chile where the outcomes of the Break-even Indicator (BEI) method, and the inverse distance weighting (IDW) method are compared to a reference model in terms of their ability to classify and provide destination for materials. The results show that BEI is superior to IDW in that it produced results closer to the reference model. The study further stated that the simulation based approached used here produced results similar yet slightly better than the BEI. Meanwhile, Ordinary kriging (OK) produced a marginally inferior result. Only results for BEI and IDW are presented here with respect to the reference model for tonnages and total Cu grade for different destinations. Proximity of a value to 1.0 indicates better performance of the method. A factor greater than 1 implies overestimation with respect to the reference model. Details of this work can be found in (Rossi & Deutsch, 2014).

Table 1: Performance of IDW and BEI models with respect to an SGS reference model. Source: (Rossi & Deutsch, 2014).

Destination code	**Tonnage w.r.t reference**		**TCu Grade w.r.t reference**	
	IDW	BEI	IDW	BEI
SAL	1.10	1.10	0.91	0.92
SME	1.16	1.09	1.06	1.00
SBA	0.18	0.45	1.15	1.01
SMR	0.50	0.43	1.36	1.01
SAS	0.55	0.87	1.02	0.95
OXA	1.29	1.13	0.85	0.93
OXB	1.16	1.98	1.08	0.98
OXL	0.44	1.49	1.54	1.41
MIX	0.52	0.71	0.90	0.78
TOTAL	1.16	1.11	0.84	0.89

In the second scenario, a numerical experiment was conducted to evaluate the effectiveness of simulation versus different estimation methods based on the losses incurred as derived from the expected loss function. The penalty coefficients for underestimating and overestimating, b_1 and b_2 respectively are given as a ratio. A ratio of 1:1 means the consequences of under or overestimating are equal. If the ratio is asymmetric (example 1:2), it means that the penalty for overestimating is higher. Only a few ratios are shown here for illustrative purposes. The results show that the simulation-based method incurred the least losses than the other methods both on average and on individual penalty considerations.

Table 2: Incurred losses of grade control methods. Source: (Vasylchuk & Deutsch, 2018).

GC				$\boldsymbol{b_1: b_2}$				
Methods	2.65:1	2:1	1.3:1	1:1	1:1.3	1:2	1:2.65	**Average**
NN	1208.4	433.3	257.0	206.8	246.6	387.3	1006.0	454.0
ID	900.6	333.3	204.2	166.4	203.9	331.8	894.2	369.3
OK	877.7	323.7	197.6	160.6	196.4	318.5	855.1	359.8
SK	861.9	316.5	192.4	156.0	190.3	307.2	820.9	349.8
Simulation	338.7	215.6	186.7	155.8	186.3	249.7	355.0	224.7

The last scenario involves the integration of machine learning techniques into grade control. The two ML algorithms used (ERBFN and SVR) were compared to other commonly used methods in the industry with respect to how their grade assignment led to the classification of the materials. The methods were Inverse distance (ID), ordinary kriging (OK), and an intelligent grade control (IGC) based on multivariate simulation. From the study, it was observed that ERBFN and SVR, in conjunction with the collocated cokriging framework outperformed traditional estimation methods and the simulation-based method. The efficient grade prediction process led to better classification and better prediction of material destinations. In Figure 4 below, a five-fold mean square error (MSER) validation is used to measure the performance of each model over ten blast holes considered. The red points represent the overall MSER for each method. From the study, it was observed that the inverse distance estimation (ID) obtained the highest MSER of 0.00888, while that of ERBFN and SVR obtained an MSER of 0.007 and 0.0075 respectively.

Finally, the extent of misclassification by the methods were evaluated and the study revealed that ERBFN with collocated cokriging (CCok) obtained the best performance in reducing misclassification. It obtained a reduction in misclassified material of 12% when compared to ID; 5.4% less than OK, 5.7% relative to IGC and 4.3% relative to CCok with SVR. Even though ERBFN-CCok obtained better results that SVR-CCok, the latter also obtained a reduction of 8% in misclassified material relative to ID and 1.12% and 1.16% to OK and IGC respectively as shown in Figure 5.

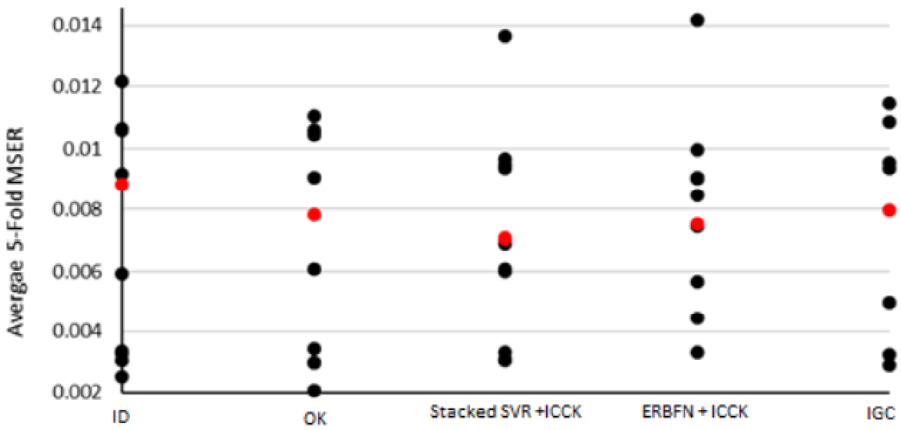

Figure 4: Mean squared error obtained from a 5-fold cross validation for each method applied (source: (Da Silva et al., 2020).

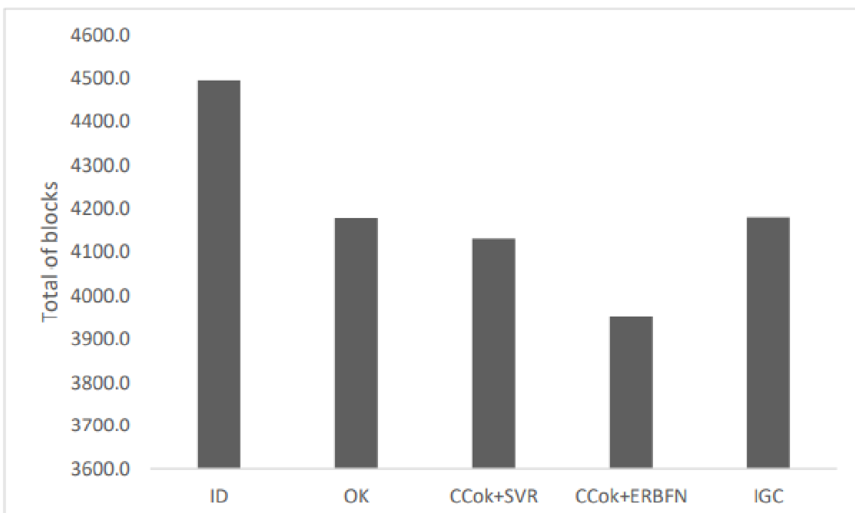

Figure 5: Total number of misclassified blocks recorded by the grade control methods (source: (Da Silva et al., 2020).

4. Conclusions

Grade control methods for classifying materials have evolved. The methods reviewed in this paper have shown that classical estimation methods can no longer be relied on in making grade control decisions. The smoothing effect and other limitations of kriging methods are a huge concern for grade control and hence do not make it an optimal method. Simulation based methods have provided good results which still make them a cutting-edge tool in grade control and is still used by most mining companies today. However, machine learning methods may become the future of grade control due to its high performance in grade prediction and significantly reducing misclassification, which is the goal of grade control.

5. Acknowledgments

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